NATIONAL TECHNICAL UNIVERSITY OF ATHENS

SCHOOL OF ELECTRICAL & COMPUTER ENGINEERING

DEPARTMENT OF SIGNALS, CONTROL AND ROBOTICS

Game Theory and Dynamic Mechanisms on Graphs

DOCTORAL DISSERTATION

ATHANASIOS RAFAIL LAGOS
Dipl. Electrical & Computer Engineer, NTUA

SUPERVISOR:
G. P. PAPAVASSILOPOULOS
Prof. Em. NTUA

ATHENS, December 2022

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Three-member Advisory Committee:

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ATHENS, December 2022

To my mother, Eleni.

Prologue

This doctoral dissertation is an outcome of research carried out in the Control and Decision Laboratory (CDL) of the School of Electrical and Computer Engineering (ECE), NTUA, under the supervision of Professor GP Papavasilopoulos, from October 2016 to June 2022.

The object of the dissertation is the study of applications of game theory in decentralized, interconnected dynamical systems, modelled with graph theoretic tools. At the same time, control protocols for finite-time consensus, secure consensus and decentralized optimization in such systems were studied.

The applications studied include, at first, the study of the effect of manipulative behaviors in social choice procedures, such as elections, and their limitation through the redesign of the network topology. The initial results of this study were presented at the AMASES 2018 conference, in Naples, while the overall results of this study were published in the scientific Journal of the Franklin Institute, Elsevier, in March 2022.

Secondly, social distancing during the outbreak of an epidemic was modeled and studied as a game among the members of a community, taking into account two different types of information available to the decision makers. This work has been submitted in the scientific journal Computer Methods and Programs in Biomedicine Update, Elsevier, and is currently under revision. In parallel, dynamic games of social distancing with asymmetric solutions were studied and the results were published in the scientific journal Dynamic Games and Applications, Springer, in October 2021. Furthermore, the effect of equity constraints on social distancing and on the spread of the epidemic was analysed and it was shown that inequality aversion affects the spread of an epidemic. This work has been submitted for publication in the scientific journal Applied Mathematics and Computation, Elsevier, and is currently under revision.

Thirdly, a stochastic consensus protocol was introduced and proved to converge almost-surely in finite time. The results of this study will be submitted for publication in the scientific journal IEEE Transactions on Circuits and Systems, in July 2022. At the same time, decentralized optimization protocols and secure communication protocols in multiagent systems were studied. A paper on decentralized optimization protocols will be published in

the scientific journal IEEE Transactions on Automatic Control in August 2022, while the study of secure communication protocols is in progress.

This dissertation includes the game theoretic applications on the study of manipulative behaviors in a social choice procedure and on the study of social distancing with different information available to the agents and the application which studies the finite-time stochastic consensus protocol.

The elaboration, writing and publication of all the aforementioned research works and the completion of my doctoral research was a long and demanding process that would not have been completed without the contribution and support of many remarkable scientists and people. I would therefore like to thank all those who contributed directly or indirectly to my research.

First of all, I would like to thank my supervisor, prof. George Papavasilopoulos, for the scientific guidance and the insightful discussions we had, especially at the beginning of my doctoral research, as well as for the freedom of choice he gave me both in terms of the content and of the way that this research was finally conducted. It is worth mentioning that I realized the value of many of his suggestions quite late, but I am now convinced about their validity and I have to thank him.

In particular, I would like to thank the lecturer of the school and member of the advisory committee Mr. Charalambos Psillakis, as well as, Dr. Ioannis Kordonis and Athanasios Gessoulis, who were my closest research collaborators during these years. Most of the scientific research I have done, including the research contained in this dissertation, has been done in collaboration with these scientists. Apart from being excellent scientists, I would like to emphasize that they are amazing people and working with them was an experience that contributed significantly both to my scientific and personal development.

Moreover, my colleagues in the Control and Decision Laboratory should be included in the people that contributed to my research. I am referring to my friend and collaborator Spyros Patmanidis, that we spent a lot of time together in the laboratory and we tried to collaborate on our common research interests, as well as Nikos Chrysanthopoulos, Nancy Zlatinski, Elena Sarri and Nasos Vassilakis with whom we worked on similar subjects, we had many interesting discussions and we shared a great working environment. I also thank the professor of Polytechnique Montreal, Mr. Roland Malhamé for our brief collaboration, our interesting discussions and his invitation for collaboration at Polytechnique Montreal, which unfortunately did not take place.

I would like also to make a special mention to two mathematicians. The first one, is the professor of the School of Applied Mathematics and Physics, Mr. Spyros Argyros, who I would like to thank for the opportunity he gave me at the beginning of my scientific career to collaborate with him and thus be able to get a glimpse of the fascinating world of theoretical mathematics and in particular of functional analysis. The second one that I would like to thank is one of my teachers in secondary education, Mr. Konstantinos Boutsikos, who instilled in me a love for mathematics, which was probably the motivation and the starting point for my research activity.

All the aforementioned people undoubtedly contributed to my research, however, that research would not be feasible if there were not some people who supported me, helped me to face all my personal problems and guaranteed me a good quality of life, so that I can continue my research. I am referring to my parents Eleni and Panagiotis, to my brother Dimitris and to all my friends who stood by me all these years. I thank them all warmly, their contribution is definitely invaluable.

ATHANASIOS RAFAIL LAGOS
December 2022

Abstract

In this thesis, game theoretic and control methods have been applied in problems arising in decentralised networked systems. Three applications have been considered. The first deals with opinion dynamics and manipulation in social networks. The second is related to the spontaneous response of a population to an epidemic outbreak through social distancing. The third introduces a stochastic consensus protocol for finite-time coordination of agents with high-order dynamics. In these three applications, the structure of the system is interconnected, the agents possess some kind of intelligence and act in a decentralised way. So, either game situations arise and the equilibria are studied or decentralised control protocols are necessary to achieve some collective goal.

In the first application, a social choice procedure is modeled as a Nash game among the agents. The agents are communicating with each other through a communication network e.g., a social network, modeled by an undirected graph and their opinions follow a dynamic rule modelling conformity. The agents' criteria for this game are describing a trade off between self-consistent and manipulative behaviors. Their best response strategies are resulting in a dynamic rule for their actions. The stability properties of these dynamics are studied. In the case of instability, which arises when the agents are highly manipulative, the stabilization of these dynamics through the design of the network topology is formulated as a constrained integer programming problem. The constraints have the form of a Bilinear Matrix Inequality (BMI), which is known to result in a nonconvex feasible set in the general case. To deal with this problem a genetic algorithm, which uses a Linear Matrix Inequality (LMI) solver during the selection procedure, is designed.

The second application deals with the choice of a population to apply social distancing, which is modeled as a Nash game where the agents determine their social interactions. The interconnections among the agents are modeled by a network. The information available to the agents plays a crucial role. Two information patterns are examined, the case that the agents know exactly the health states of their neighbors and the case they have only statistical information for the global prevalence of the epidemic. The agents are considered to be myopic, and thus, the Nash equilibria of static games for each day are studied. The Nash equilibria are characterized and algorithms are introduced to compute them. Moreover, the

effects of the network structure, the virus transmissibility, the number of vulnerable agents, the health care system capacity and the information quality (fake news) are examined through simulations.

In the third application a novel stochastic minimum-maximum consensus protocol is introduced and analyzed. The stochastic mixing and the low computational effort make this protocol well suited for secure consensus in cyber-physical systems, composed by autonomous agents with limited resources. It is proven that the protocol converges almost-surely in finite time. Furthermore, the application of the stochastic consensus protocol coupled with a finite-time control law on a distributed system of agents with high-order dynamics is considered and it is proven that the agents' states converge in finite time. Finally, simulations showing the efficiency of this decentralised finite-time control scheme on double-integrator agents are presented.

Keywords: Networked Systems; Multiagent Systems; Games on Networks; Nash Games; Opinion dynamics; Epidemics on Networks; Finite-time Consensus; Network Topology Design; Genetic Algorithms; Stochastic Algorithms; Information Patterns.

Abstract in Greek

Σε αυτή τη διατριβή, μελετώνται εφαρμογές μεθόδων της θεωρίας παιγνίων και του αυτομάτου ελέγχου σε προβλήματα που ανακύπτουν σε αποκεντρωμένα διασυνδεδεμένα συστήματα. Συγκεκριμένα, εξετάζονται τρεις εφαρμογές. Η πρώτη εφαρμογή αφορά στην διάδοση απόψεων και τη χειραγώγηση σε κοινωνικά δίκτυα. Η δεύτερη σχετίζεται με την αυθόρμητη ανταπόκριση ενός πληθυσμού στο ξέσπασμα μιας επιδημίας μέσω της εφαρμογής κοινωνικής αποστασιοποίησης. Στην τρίτη εφαρμογή εισάγεται ένα πρωτόκολλο στοχαστικής συνεννόησης (consensus) για τον συντονισμό πρακτόρων με δυναμική υψηλής τάξης σε πεπερασμένο χρόνο. Σε αυτές τις τρεις εφαρμογές, η δομή του συστήματος είναι διασυνδεδεμένη, οι πράκτορες διαθέτουν κάποιο είδος νοημοσύνης και ενεργούν με αποκεντρωμένο τρόπο. Έτσι, είτε προκύπτουν καταστάσεις παιγνίου και μελετώνται τα σημεία ισορροπίας είτε απαιτούνται αποκεντρωμένα πρωτόκολλα ελέγχου για την επίτευξη κάποιου συλλογικού στόχου.

Στην πρώτη εφαρμογή, μια διαδικασία κοινωνικής επιλογής μοντελοποιείται ως παίγνιο Ναση μεταξύ των πρακτόρων. Οι παίκτες επικοινωνούν μεταξύ τους μέσω ενός δικτύου επικοινωνίας, π.χ., ενός κοινωνικού δικτύου, μοντελοποιημένου από ένα μη κατευθυνόμενο γράφο και οι απόψεις τους έχουν δυναμικές που μοντελοποιούν το φαινόμενο της σύγκλισης λόγω συμμόρφωσης προς τις απόψεις των γειτόνων. Τα κριτήρια των παικτών για αυτό το παίγνιο εκφράζουν την διελκυστίνδα της επιλογής μεταξύ αυτοσυνεπών και χειραγωγητικών συμπεριφορών. Οι βέλτιστες αποκρίσεις τους καταλήγουν σε έναν δυναμικό κανόνα για τις ενέργειές τους. Μελετάται η ευστάθεια αυτών των δυναμικών. Στην περίπτωση της αστάθειας, η οποία προκύπτει όταν οι παίκτες είναι εξαιρετικά χειραγωγητικοί, η σταθεροποίηση αυτής της δυναμικής μέσω του σχεδιασμού της τοπολογίας του δικτύου διατυπώνεται ως ένα πρόβλημα ακέραιου προγραμματισμού. Οι περιορισμοί έχουν τη μορφή Διγραμμικής Ανισότητας Πινάκων (BMI), η οποία είναι γνωστό ότι οδηγεί σε ένα μη κυρτό σύνολο εφικτών λύσεων, στη γενική περίπτωση. Για την αντιμετώπιση αυτού του προβλήματος προτείνεται ένας γενετικός αλγόριθμος, ο οποίος χρησιμοποιεί λογισμικό επίλυσης Γραμμικών Ανισοτήτων Πίνακων (LMI) κατά τη διαδικασία επιλογής της επόμενης γενιάς.

Η δεύτερη εφαρμογή ασχολείται με την επιλογή ενός πληθυσμού να εφαρμόσει κοινωνική αποστασιοποίηση, η οποία διατυπώνεται ως παίγνιο Ναση, όπου οι παίκτες καθορίζουν τις κοινωνικές τους αλληλεπιδράσεις. Οι διασυνδέσεις μεταξύ των παικτών μοντελοποιούνται από ένα γράφο που περιγράφει το δίκτυο των κοινωνικών τους επαφών. Η πληροφορία που είναι διαθέσιμη στους παίκτες έχει κρίσιμο ρόλο στο παίγνιο αυτό. Εξετάζονται δύο σενάρια διαθέσιμης πληροφορίας, η περίπτωση που οι παίκτες γνωρίζουν ακριβώς την κατάσταση της υγείας των γειτόνων τους και η περίπτωση που διαθέτουν μόνο στατιστική πληροφορία για την συνολική διάδοση της επιδημίας. Οι παίκτες θεωρούνται μυωπικοί και έτσι μελετώνται οι ισορροπίες Ναση των στατικών παιγνίων που προκύπτουν για κάθε ημέρα. Οι ισορροπίες Nash χαρακτηρίζονται και εισάγονται αλγόριθμοι για τον υπολογισμό τους. Επιπλέον, μέσω προσομοιώσεων εξετάζονται οι επιπτώσεις της δομής του δικτύου, της μεταδοτικότητα του ιού, του αριθμού των ευάλωτων παικτών, των δυνατοτήτων του συστήματος υγειονομικής περίθαλψης και της ποιότητας των πληροφοριών (fake news).

Στην τρίτη εφαρμογή εισάγεται και αναλύεται ένα νέο στοχαστικό πρωτόκολλο συνενόησης, που βασίζεται στη στοχαστική μίξη της ελάχιστης και της μέγιστης τιμής των γειτόνων κάθε πράκτορα. Η στοχαστική μίξη και η χαμηλή υπολογιστική προσπάθεια καθιστούν αυτό το πρωτόκολλο κατάλληλο για ασφαλή συναίνεση σε κυβερνοφυσικά (cyberphysical) συστήματα, που αποτελούνται από αυτόνομους πράκτορες με περιορισμένους πόρους. Αποδεικνύεται ότι το πρωτόκολλο συγκλίνει σχεδόν σίγουρα σε πεπερασμένο χρόνο. Επιπλέον, εξετάζεται η εφαρμογή του πρωτοκόλλου στοχαστικής συνενόησης σε συνδυασμό με έναν νόμο ελέγχου πεπερασμένου χρόνου σε ένα κατανεμημένο σύστημα πρακτόρων με δυναμική υψηλής τάξης και αποδεικνύεται ότι οι καταστάσεις των πρακτόρων συγκλίνουν σε πεπερασμένο χρόνο. Τέλος, παρουσιάζονται προσομοιώσεις που δείχνουν την αποτελεσματικότητα αυτού του αποκεντρωμένου σχήματος ελέγχου πεπερασμένου χρόνου σε πράκτορες με δυναμικές διπλού ολοκληρωτή.

Λέξεις Κλειδιά: Θεωρία Παιγνίω· Πολυπρακτορικά Συστήματα· Δυναμικές Απόψεων· Δυναμικές Επιδημιών· Αποκεντρωμένα Πρωτόκολλα Ελέγχου

Summary in Greek

Αντικείμενο της παρούσας διδακτορικής διατριβής είναι η μελέτη προβλημάτων που εμφανίζονται σε πολυπρακτορικά συστήματα, με έμφαση σε προβλήματα αποκεντρωμένης λήψης αποφάσεων.

Η μεγάλη εξάπλωση σύνθετων αποκεντρωμένων, αλλά διασυνδεδεμένων, συστημάτων σε πολλά πεδία, όπως οι τηλεπικοινωνίες, η ενέργεια, η επιδημιολογία, τα κοινωνικά δίκτυα κ.α., είναι χαρακτηριστικό της εποχής μας. Η ταχεία ανάπτυξη των τηλεπικοινωνιακών τεχνολογιών, των ενσωματωμένων συστημάτων και των μικροϋπολογιστών συνέβαλε σε αυτό, καθώς πλέον οι περισσότερες συσκευές διαθέτουν δυνατότητες διασύνδεσης, αλλά και στοιχειώδη ευφυϊα, υπό την έννοια των διαθέσιμων υπολογιστικών πόρων, και ως εκ τούτου μπορούν να συνεργαστούν και να επιτελέσουν σύνθετες λειτουργίες σε επίπεδο συστήματος.

Εξαιτίας αυτού του μετασχηματισμού της πλειονότητας των συστημάτων σε αποκεντρωμένα και περίπλοκα συστήματα με δικτυακή τοπολογία ανακύπτουν αρκετές νέες προκλήσεις και προβλήματα όσον αφορά στην σχεδίασή τους και την λειτουργία τους. Τέτοιες προκλήσεις είναι η αποκεντρωμένη φύση του συστήματος, η πολυπλοκότητα των αλληλεπιδράσεων μεταξύ των λειτουργικών στοιχείων του, η ανομοιογένεια των στοιχείων αυτών, η ασυμμετρία της διαθέσιμης πληροφορίας σε αυτά κ.α.

Προς αντιμετώπιση αυτών των προκλήσεων, νέα αλλά και ήδη υπάρχοντα θεωρητικά μοντέλα και εργαλεία επιστρατεύονται για την μοντελοποίηση και την ανάλυση αυτών των συστημάτων. Συγκεκριμένα, θεωρητικά εργαλεία από τις περιοχές του αυτομάτου ελέγχου, της θεωρίας γράφων και δικτύων, της θεωρίας παιγνίων και της βελτιστοποίησης εφαρμόζονται κατα κόρον σε αυτήν την ενεργή ερευνητική περιοχή.

Στη συγκεκριμένη διατριβή, οι μελέτες εστιάζουν στην μοντελοποίηση και ανάλυση πολυπρακτορικών συστημάτων. Τα συστήματα αυτά χαρακτηρίζονται από ευφυείς κόμβους, των οποίων η συμπεριφορά μοντελοποιείται από κάποια κατάλληλη δυναμική. Παραδείγματα τέτοιων συστημάτων είναι σμήνη από ρομπότ, δίκτυα αισθητήρων, μικροδίκτυα (ενέργειας), κοινωνικά δίκτυα κ.α. Βασικό ζήτημα των μελετών σε πολυπρατκορικά συστήματα, οι οποίες περιλαμβάνονται στην διατριβή, είναι οι αποκεντρωμένες διαδικασίες

λήψης αποφάσεων, για την ανάλυση των οποίων επιστρατεύονται τεχνικές της θεωρίας παιγνίων και του αυτομάτου ελέγχου. Ουσιαστικά, τα κοινά αυτά θεωρητικά εργαλεία αποτελούν τον συνδετικό κρίκο ανάμεσα στα μέρη της διατριβής, τα οποία αφορούν εφαρμογές σε διαφορετικά πεδία.

Οι εφαρμογές που παρουσιάζονται στα τρία διακριτά μέρη της διατριβής αφορούν:

- στην σχεδίαση τοπολογίας δικτύου με στόχο τον περιορισμό χειραγωγητικών συμπεριφορών κατά τη διάρκεια μιας διαδικασίας κοινωνικής επιλογής
- στην μοντελοποίηση της κοινωνικής αποστασιοποίησης κατα τη διάρκεια μιας επιδημίας ως παίγνιο μεταξύ των μελών ενός πληθυσμού, με ιδιαίτερη έμφαση στον ρόλο της διαθέσιμης πληροφορίας
- στην ανάλυση ενός στοχαστικού πρωτοκόλλου ομοφωνίας, το οποίο αποδεικνύεται ότι συγκλίνει σχεδόν-βέβαια σε πεπερασμένο χρόνο, και στην σχεδίαση ενός αποκεντρωμένου νόμου ελέγχου για πολυπρακτορικά συστήματα με στόχο την συνεννόηση σε πεπερασμένο χρόνο

Στις ακόλουθες υποενότητες αυτού του εισαγωγικού κεφαλαίου, παρουσιάζονται συνοπτικά τα μοντέλα, οι τεχνικές και τα αποτελέσματα της έρευνας που πραγματοποιήθηκε επί των συγκεκριμένων εφαρμογών.

Μέρος 1: Σχεδίαση τοπολογίας δικτύου με στόχο τον περιορισμό χειραγωγητικών συμπεριφορών κατά τη διάρκεια μιας διαδικασίας κοινωνικής επιλογής

Η πρώτη θεματική ενότητα που μελετήθηκε σχετίζεται με μοντέλα διάδοσης απόψεων σε πλήθη και λήψης αποφάσεων σε μια διαδικασία κοινωνικής επιλογής (π.χ. εκλογές), όπου οι παίκτες έχουν χειραγωγητικές συμπεριφορές. Εξαιτίας των συμπεριφορών αυτών οι δυναμικές των παικτών είναι ασταθείς σε ορισμένες περιπτώσεις. Ως εκ τούτου, μελετήθηκε η επανασχεδίαση της τοπολογίας του δικτύου με στόχο την αποφυγή των ασταθειών αυτών.

Συγκεκριμένα, για τα μοντέλα διάδοσης απόψεων και λήψης αποφάσεων έναυσμα στάθηκε η ερευνητική εργασία [38], που μελετά ένα παίγνιο όπου οι παίκτες έχουν χειραγωγητικές συμπεριφορές. Στην παρούσα μελέτη επαυξήσαμε το μοντέλο της εργασίας αυτής θεωρώντας πως οι απόψεις των παικτών εξελίσσονται δυναμικά. Οι δυναμικές των απόψεων των παικτών μοντελοποιούν το φαινόμενο της επιρροής από τις απόψεις των γειτόνων τους και της σύγκλισης σε κάποιο σημείο συμφωνίας. Οι πράξεις των παικτών προκύπτουν ως οι βέλτιστες αποκρίσεις τους (best responses) στις πράξεις των γειτόνων τους, με βάση τα ατομικά τους κριτήρια, οι οποίες οδηγούν σε ένα σημείο ισορροπίας Nash. Τα κριτήρια των παικτών μοντελοποιούν αφενός την συνέπεια των πράξεών τους με τις απόψεις τους και αφετέρου την επιθυμία τους να επηρέασουν το αποτέλεσμα της διαδικασίας κοινωνικής επιλογής προς όφελός τους χειραγωγώντας τους γείτονές τους.

Από τις συζευγμένες δυναμικές των απόψεων και των πράξεων των παικτών, που προκύπτουν όπως περιγράφηκε παραπάνω, καταλήγουμε σε ένα σύστημα δευτέρας τάξεως. Η ευστάθεια του συγκεκριμένου συστήματος μελετήθηκε και δόθηκαν ικανές συνθήκες ώστε οι δυναμικές να είναι ευσταθείς. Οι συνθήκες αυτές διατυπώνονται ως μια ανισότητα πινάκων, τα ορίσματα των οποίων εξαρτώνται από τις παραμέτρους που μοντελοποιούν πόσο χειραγωγητικοί είναι οι παίκτες και από την τοπολογία του δικτύου.

Εξαιτίας αυτής της εξάρτησης η τοπολογία του δικτύου θεωρήθηκε ως σχεδιαστική παράμετρος με στόχο να αποφευχθούν οι αστάθειες στις δυναμικές των παικτών - οι οποίες μπορούν να ερμηνευθούν ως φαινόμενα κοινωνικής πόλωσης ή και σύγκρουσης. Έτσι διατυπώθηκε ένα πρόβλημα σχεδίασης της τοπολογίας του δικτύου ώστε οι δυναμικές να είναι ευσταθείς. Το πρόβλημα αυτό είναι πρόβλημα συνδυαστικής βελτιστοποίησης με περιορισμούς μη κυρτούς ως προς τις μεταβλητές απόφασεις. Αυτό αποδείχθηκε ανάγοντας τους περιορισμούς σε μία διγραμμική ανισότητα πινάκων (Bilinear Matrix Inequality), που είναι μη κυρτή. Ως τέτοιο το συγκεκριμένο πρόβλημα βελτιστοποίησης είναι ένα δύσκολα επιλύσιμο πρόβλημα και για αυτό τον λόγο αναπτύχθηκε ένας γενετικός αλγόριθμος ο οποίος σε συνδυασμό με ένα λογισμικό που επιλύει αποτελεσματικά γραμμικές ανισότητες πινάκων (Linear Matrix Inequalities), βρίσκει ευριστικά κάποια υποβέλτιστη λύση, η οποία ωστόσο είναι εφικτή, δηλαδή λύνει το πρόβλημα της ευσταθιοποίησης των δυναμικών και μπορεί να βρίσκεται κοντά στην βέλτιστη λύση.

Εν τέλη, για την εξέταση του προτεινόμενου αλγορίθμου πραγματοποιήθηκαν πλήθος προσομοιώσεων σε διάφορες τοπολογίες γράφων. Από τις προσομοιώσεις αυτές παρατηρήθηκε αφενός ότι ο αλγόριθμος επιστρέφει εφικτές λύσεις σχετικά κοντά στην αρχική τοπολογία και συνεπώς κοντά στο βέλτιστο. Αφετέρου, ότι σε ορισμένες περιπτώσεις, οι τοπολογίες που προκύπτουν είναι μη συνδεδεμένες, που σημαίνει ότι αποκλείει κοινωνικά κάποιους χειραγωγητικούς παίκτες, το οποίο ωστόσο μπορεί να μην είναι κοινωνικά αποδεκτό. Αυτό επιλύεται με την εισαγωγή κάποιων νέων γραμμικών περιορισμών στο πρόβλημα βελτιστοποίησης, οι οποίοι δεν επηρεάζουν την πολυπλοκότητά του και συντελούν στην εύρεση υποβέλτιστων αλλά συνδεδεμένων τοπολογιών.

Μέρος 2: Παίγνια Κοινωνικής Αποστασιοποίησης κατά τη διάρκεια μιας Επιδημίας: Τοπική έναντι Στατιστικής Πληροφόρια

Η δεύτερη θεματική ενότητα που μελετήθηκε σχετίζεται με το ζήτημα της κοινωνικής αποστασιοποίησης κατά την διάρκεια μιας επιδημιάς, το οποίο μοντελοποιείται ως παίγνιο μεταξύ των μελών μιας κοινότητας. Ιδιαίτερα χαρακτηριστικά της μελέτης αυτής είναι πως χρησιμοποιήθηκε ένα ατομοκεντρικό μοντέλο (agent-based model), δόθηκε σημασία στην μοντελοποίηση της δικτυακής φύσης των ανθρώπινων συναναστροφών και στις συνεπαγόμενες τοπικές αλληλεπιδράσεις μεταξύ τους, καθώς και στον ρόλο της διαθέσιμης πληροφορίας κατά την διαδικασία λήψης αποφάσεων περί της εφαρμογής κοινωνικής αποστασιοποίησης.

Στο μοντέλο που χρησιμοποιήθηκε οι συναναστροφές των ατόμων μοντελοποιούνται με τη χρήση ενός γράφου. Η κατάσταση υγείας, η οποία διακρίνει αν το άτομο δεν έχει νοσήσει, αν νοσεί ή αν έχει αναρρώσει, μοντελοποιείται με τη χρήση δύο μεταβλητών κατάστασης που εξελίσσονται δυναμικά επηρεαζόμενες από τις συναναστροφές του κάθε ατόμου. Αξίζει να αναφερθεί πως με κατάλληλη επιλογή της πιθανότητας μετάδοσης της νόσου από άτομο σε άτομο κατά την συναναστροφή τους το ατομοκεντρικό αυτό μοντέλο που χρησιμοποιούμε προσεγγίζει το γνωστό SIR μοντέλο που χρησιμοποιείται ευρέως στην επιδημιολογία.

Όσον αφορά στην μοντελοποίηση της κοινωνικής αποστασιοποίησης ως παίγνιο θεωρούμε τα ακόλουθα. Οι μεταβλητές απόφασης του κάθε ατόμου αποτελούν ένα διάνυσμα που εκφράζει την ένταση της συναναστροφής του με κάθε έναν από τους γείτονές του επί του γράφου κάθε μέρα της εξέλιξης της επιδημίας. Τα κριτήρια των ατόμων εκφράζουν αφενός τις απολαβές (απόλαυση) που κάποιος εισπράτει από τις κοινωνικές του συναναστροφές και αφετέρου τον φόβο του ενδεχομένου να νοσήσει εξαιτίας των συναναστροφών αυτών. Οι πράξεις των ατόμων καθορίζονται τοπικά ως οι βέλτιστες αποκρίσεις τους στις πράξεις των γειτόνων τους, με βάση τα κριτήριά τους, θεωρώντας πως τα άτομα έχουν κίνητρο να παίξουν σε κάποιο σημείο ισορροπίας Nash. Ιδιαίτερο ρόλο στις αποφάσεις των ατόμων παίζει η πληροφορία που έχουν στη διάθεσή τους και επ αυτού εξετάζονται δύο σενάρια, αυτό της τοπικής και αυτό της στατιστικής πληροφορίας.

Στο πρώτο σενάριο, αυτό της τοπικής πληροφορίας, θεωρείται πως τα άτομα γνωρίζουν τις ακριβείς καταστάσεις υγείας των γειτόνων του κάθε χρονική στιγμή και μπορούν να επιλεξούν ορθολογικά ποιούς από αυτούς θα δούν αναλόγως τον κίνδυνο που διατρέχουν. Σε αυτήν την περιπτωση αποδεικνύεται πως τα σημεία ισορροπίας Nash βρίσκονται στο σύνορο του χώρου των στρατηγικών, δηλαδή κάθε άτομο διαλέγει είτε να συναστραφεί πλήρως με κάποιον είτε να μην τον συναντήσει καθόλου. Προτείνεται επίσης ένας κατανεμημένος αλγόριθμος που συγκλίνει σε σημείο ισορροπίας Nash του παιγνίου.

Στο δεύτερο σενάριο, αυτό της στατιστικής πληροφορίας, θεωρείται πως τα άτομα γνωρίζουν μόνο κάποιους στατιστικούς δείκτες για την εξάπλωση της επιδημίας, το ποσοστό των νοσούντων και των αναρρωσάντων. Σε αυτήν την περίπτωση λαμβάνονται κάποιες επιπλέον υποθέσεις διότι η έλλειψη πληροφορίας δεν επιτρέπει την λήψη τόσο συγκεκριμένων αποφάσεων όπως στο άλλο σενάριο. Οι υποθέσεις αυτές είναι πως το άτομο επιλέγει μια κοινή ένταση συναναστροφής με όλους τους γείτονές τους καθώς δεν μπορεί να διακρίνει τις καταστάσεις υγείας τους και πως τα άτομα αγνοούν τις όποιες αλληλοσυσχετίσεις μεταξύ των καταστάσεων υγείας τους. Με βάση αυτές τις υποθέσεις αποδεικνύεται και σε αυτήν την περίπτωση πως τα σημεία ισορροπίας Nash βρίσκονται στο σύνορο του χώρου των στρατηγικών, δηλαδή κάθε άτομο διαλέγει είτε να συναστραφεί πλήρως τους γείτονες του είτε να απομονωθεί. Και σε αυτήν την περίπτωση, προτείνεται κατάλληλος κατανεμημένος αλγόριθμος που συγκλίνει σε σημείο ισορροπίας Nash του παιγνίου.

Εν συνεχεία, παρουσιάζονται προσομοιώσεις που καταδεικνύουν διεξοδικά τα χαρακτηριστικά των δύο σεναρίων του παιγνίου κοινωνικής αποστασιοποίησης. Βασικό συμπέρασμα αποτελεί πως και στις δύο περιπτώσεις η εξάπλωση της επιδημίας περιορίζεται από τις αποκεντρωμένες αποφάσεις κοινωνικής αποστασιοποίησης που λαμβάνουν τα άτομα, αλλά στην περίπτωση της στατιστικής πληροφορίας η κοινωνική αποστασιοποίηση που εφαρμόζεται είναι πολύ πιο αυστηρή λόγω της έλλειψης γνώσης για την κατάσταση υγείας των επαφών των ατόμων. Επίσης, είναι αξιοσημείωτο ότι, σε επίπεδο απολαβών, το κόστος αυτής της πιο αυστηρής κοινωνικής αποστασιοποίησης το πληρώνουν τα πιο ευπαθή μέλη της κοινότητας, καθώς αυτοί φοβούνται περισσότερο με αποτέλεσμα να στερούνται τις κοινωνικές τους συναναστροφές.

Παράλληλα, παρουσιάζονται πολλές επιμέρους προσομοιώσεις μέσω των οποίων μελετάται η επίδραση διαφόρων παραμέτρων των μοντέλων στην εξάπλωση της επιδημίας και στην συμπεριφορά των παικτών. Συγκεκριμένα, εξετάζονται διαφορετικές τοπολογίες του δικτύου κοινωνικών συναναστροφών, διαφορετικοί βαθμοί (μέσος αριθμός γειτόνων), διαφορετική μεταδοτικότητα του ιού, διαφορετικά ποσοστά ευπαθών ατόμων στην κοινωνία. Εξετάζονται, τέλος, το σενάριο η ευπάθεια των ατόμων να εξαρτάται από την εξάπλωση της επιδημίας, όπως μπορεί να συμβεί στην περίπτωση ενός σχετικά ανεπαρκούς συστήματος δημόσιας υγείας και το σενάριο διάδοσης ψευδούς στατιστικής πληροφορίας για την εξάπλωση της επιδημίας.

Μέρος 3: Στοχαστικό Πρωτόκολλο Συνεννόησης που συγκλίνει Σχεδόν-Βέβαια σε Πεπερασμένο Χρόνο

Η τρίτη θεματική ενότητα αφορά στην μελέτη ενός νέου στοχαστικού πρωτοκόλλου συνεννόησης (consensus) που συγκλίνει σχεδόν-βέβαια σε πεπερασμένο χρόνο, καθώς και στην σχεδίαση ενός κατανεμημένου νόμου ελέγχου για πολυπρακτορικά συστήματα, όπου οι δυναμικές των πρακτόρων μπορούν να περιγραφούν ως αλυσίδες ολοκληρωτών, ώστε να επιτυγχάνεται σύγκλιση των δυναμικών σε κάποιο κοινό σημείο σε πεπερσμένο χρόνο.

Χαρακτηριστικό του προτεινόμενου πρωτοκόλλου είναι ότι η τυχαιότητα εισάγεται σκόπιμα, ούτως ώστε το πρωτόκολλο να είναι ασφαλές έναντι πιθανής προσπάθειας υποκλοπής της τιμής σύγκλισης. Επίσης, χαρακτηριστικό του προτεινόμενου νόμου ελέγχου είναι πως χρησιμοποιούνται μόνο δείγματα των εξόδων των γειτονικών πρακτόρων, με αποτέλεσμα εξοικονόμηση εύρους ζώνης και ενέργειας. Τέλος, το στοχαστικό πρωτόκολλο συνεννόησης και ο νόμος ελέγχου δουλεύουν σε δίκτυα με μεταβαλλόμενη τοπολογία. Βασική υπόθεση είναι αυτή η μεταβαλλόμενη τοπολογία να είναι ομοιόμορφα, από κοινού, ισχυρά συνδεδεμένη (Uniformly Jointly Strongly Connected).

Το στοχαστικό πρωτόκολλο που εισάγουμε στην συγκεκριμένη μελέτη είναι ουσιαστικά μία στοχαστική μίξη (κυρτός συνδυασμός με τυχαίους συντελεστές) της ελάχιστης και της μέγιστης τιμής των εξόδων των γειτόνων κάθε πράκτορα. Οι συντελεστές του κυρτού αυτού συνδυασμού εξαρτώνται από κάποιες τυχαίες μεταβλητές και υποθέτουμε ότι έχουν θετικές και μη φθίνουσες στον χρόνο πιθανότητες να επιλέγουν την ελάχιστη (ή ισοδύναμα την μέγιστη) τιμή.

Η ιδέα της απόδειξης της σύγκλισης του πρωτοκόλλου αυτού σε πεπερασμένο χρόνο είναι να δείξουμε πως το ενδεχόμενο οι πράκτορες να επιλέγουν διαδοχικά για ικανοποιητικό διάστημα την ελάχιστη (ή την μέγιστη) τιμή των εξόδων των γειτόνων τους - με αποτέλεσμα το πρωτόκολλό να γίνει μια απλή διάδοση του ελαχίστου (ή του μεγίστου) και να συγκλίνει - είναι κάτω φραγμένη. Ποιοτικά, αφού το ενδεχόμενο αυτό έχει θετική πιθανότητα αν το πρωτόκολλο εφαρμοστεί για μεγάλο χρονικό διάστημα κάποια στιγμή θα πραγματοποιηθεί σχεδόν-βέβαια.

Στη συνέχεια, εισάγεται ο κατανεμημένος νόμος ελέγχου για πράκτορες με δυναμικές που περιγράφονται ως αλυσίδες ολοκληρωτών. Ο νόμος αυτός δέχεται τα δείγματα των εξόδων των γειτόνων και με τη χρήση κάποιων κατάλληλων, επαρκείς φορές διαφορίσιμων, σιγμοειδών συναρτήσεων ορίζει κάποιες συνεχείς μεταβλητές σφάλματος. Έπειτα, με την χρήση ενός κατάλληλου νόμου ελέγχου για αλυσίδες ολοκληρωτών που εισάγεται στην ερευνητική εργασία [14] οδηγεί τις δυναμικές των μεταβλητών σφάλματος στο μηδέν σε πεπερασμένο χρόνο. Σε συνδυασμό με το στοχαστικό πρωτόκολλο συνεννόησης εφαρμοσμένο στα δείγματα των εξόδων των γειτόνων, αυτός ο νόμος ελέγχου οδηγεί τις αρχικές δυναμικές των πρακτόρων σε κάποια κοινή τιμή σε πεπερασμένο χρόνο.

Η αποτελεσματικότητα του συγκεκριμένου στοχαστικού πρωτοκόλλου σε συνδυασμό με τον κατανεμημένο νόμο ελέγχου παρουσιάζεται μέσω προσομοιώσεων, από τις οποίες συνάγουμε πως τις περισσότερες φορές οι δυναμικές συγκλίνουν επαρκώς γρήγορα για μικρό αριθμό πρακτόρων.

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Nomenclature

LMI	Linear Matrix Inequality
BMI	Bilinear Matrix Inequality
M2M	Machine to Machine
UAVs	Unmanned Aerial Vehicles
UGVs	Unmanned Ground Vehicles
UUVs	Unmanned Underwater Vehicles
MASs	Multi-Agent Systems
LANs	Local Area Networks
NE	Nash Equilibrium
NP	Nondeterministic Polynomial time
SIR	Susceptible-Infected-Recovered
UJSC	Uniformly Jointly Strongly Connected

Chapter 1

Introduction

1.1 The emergence of a networked world

In the last decades, a huge progress has been carried out in computing and communication technologies, automation and digitalization, which has drastically transformed many areas of human activity. The changes brought out in the industry and society have been characterized as the 4^{th} Industrial Revolution, since their momentum has significantly affected many industrial sectors and many domains of human social life.

A core characteristic of this revolution is the emergence of networked systems in place of traditional stand-alone systems and facilities. Since the development of the Internet, that was indisputably a most successful implementation of a network of computer networks, the advances in microcomputing and embedded systems, which enhanced the capabilities of small devices and the breakthroughs in Machine to Machine (M2M) communication contributed to the emergence of a highly networked world.

The transformation of industrial production and energy facilities into large interconnected systems, the access and active participation of the majority (63%) of the global population to the internet and the emergence of networks of interconnected devices in almost every sector of our daily life e.g., smart phones, smart buildings, smart cars etc, clearly indicate the significance of network science in our era.

There is a large variety of applications of engineered networked systems that are currently well-established and widely applied. Some indicative examples are the telecommunication networks, computer networks, power distribution networks, cyberphysical systems, networks of mobile agents, sensor networks and social networks.

The fast development of network science was a natural outcome of that technological revolution. Except from the study of the various types of networks that humans have engineered to communicate, interact and facilitate their activities, network science has also contributed to the better understanding of physical phenomena. The modelling, study and analysis of several physical systems as networked systems have offered valuable insights about their functioning and their intrinsic properties. For example, applications of models that take under account the networked structure of complex systems exist in physics, material science, biology, epidemiology and sociology.

1.1.1 Features, challenges and problems of networked systems

The majority of networked systems share some common features. Such features are the spatial distribution of their nodes (agents, subsystems, stations), the interconnection between them, the information exchange and/or the exchange of some other commodity, the local or global interactions between the nodes and their interdependence. Another feature, not common for all networked systems yet worth to be mentioned, is the inhomogeneity of the nodes or of the relations between them.

These features of networked systems make their analysis very challenging. So, a detailed mathematical formulation is necessary for their study. For this reason, networks are usually considered as a set of nodes and a set of communication links between these nodes. This abstract representation is the concept of a graph. Graph theory has contributed to the analysis of networked systems through its notions, such as connectivity, reachability, coverage etc, its mathematical formulation e.g., algebraic and spectral graph theory and its results.

The design, modelling and analysis of networked systems, in many cases with the use of graph theoretic methods, aims in achieving important goals for the system architecture and functionality, such as scalability, resilience, security and efficiency of its overall operation. Almost all of these problems are very challenging, in many cases their tractability depends on the specific application and if they are solvable the solution is usually computationally intensive. However, even more delicate problems have been arisen recently in this field, since the nodes obtained some kind of intelligence, that is increased computing capabilities. Such problems is the decentralised coordination, distributed optimisation, distributed estimation and control, consensus and formations.

1.2 Multiagent systems

A special category of networked systems where the nodes have computing capabilities and individual behavior described by some dynamics are called multiagent systems and they are of significant importance in modern engineering and other applications. Some typical examples of multiagent systems are the following:

- Networks of autonomous agents, such as Unmanned Aerial Vehicles (UAVs), Unmanned Ground Vehicles (UGVs) and Unmanned Underwater Vehicles (UUVs)
- Computer networks
- Sensor networks
- Microgrids
- Social networks

The problems of coordination and implementation of several complex operations, such as optimisation, in a distributed and decentralised way are the main topic of many studies on multiagent systems. The books of M.Mesbahi & M.Egerstedt [85] and M.Ren & Y.Chao [98], give a thorough description of the theoretic tools and the current advances and results in this area. However, in many cases, the nodes of these systems, called agents, except from applying a predefined protocol to achieve an operation they can also take decisions in a rational and decentralised way.

1.3 Decentralised decision making in Multiagent systems

The issue of decentralised decision making is a very important and challenging aspect of many multiagent systems, especially when humans are involved in the decision loop. For the study of such decision making procedures the use of game theoretic models and tools is a well-established and powerful approach.

In the field of game theory, the decentralised nature of the decision process, the possible conflicts of interests and the lack of access to the same information are not novel ideas. In contrast, these issues are essential, especially in the case of noncooperative games. The fact that multiagent systems are also characterised by decentralization, asymmetry on the available information during the decision making procedure and, in some cases, conflicting interests among the agents, makes game theoretical models very appealing for the analysis of such systems.

1.3.1 Applications of game theoretic models

Due to that fact, the use of game theory in the analysis of multiagent systems is an emerging research area. Driven by the technological developments, which resulted in the emergence of many networked systems, and some major events of our era (pandemic, political turmoil, wars), when the population was concerned to take decentralised decisions over important issues, there is a variety of applications of that research area the last years.

Following we provide some examples of multiagent systems, where game theoretic models have been used:

- Computer networks, telecommunication networks & cyberphysical systems: Important issues that may deal either with the confrontation of a malicious attacker, such as cybersecurity and privacy [121],[32], or with energy saving and efficiency improvements, through distributed computing on the cloud, are applications where game theoretic modelling is thriving [24].

- Power distribution networks: The enhancement of the power distribution network with communication and computing infrastructures resulted in the smart grid, which is a large interconnected system with nodes possessing some kind of intelligence. So, game theoretic models have emerged in problems related to the interactions between microgrids or the demand side management [104].

- Social networks: The study of human decision making was the raison d' être for game theory. So, the analysis of the behavior of the users of a social network, the dynamics of their opinions during advertising or marketing campaigns, the clustering of the users and their privacy issues are applications well suited for game theoretic models [108].

- Epidemics: The response of a population of interconnected agents to an epidemic outbreak can be characterised as a game situation, where spontaneous reactions, application of social distancing, vaccination, respect of the government' s instructions and spread of misinformation are issues useful to be studied in a game theoretic setting [22],[62].

For a more complete and detailed description of game theoretic applications in engineered networked systems we refer to the book of D.Bauso [12].

1.4 Outline of the dissertation

This doctoral dissertation deals with several problems in the area of decentralised networked systems and focuses more in the game theoretic applications in such systems. Specifically three distinct problems have been studied and presented in the following chapters.

In Chapter 2, a game theoretic model dealing with the existence of manipulative behaviors in a social choice procedure is presented. These manipulative behaviors are considered to arise in the process of opinions spreading over a social network and in some cases they lead to instabilities in the dynamics of the agents' actions. For this reason, a topology design problem is studied, so as the network administrator to be able to contain the instabilities arisen from intense manipulation with a proper redesign of the communication structure among the agents. This topology design problem is formulated as a nonconvex integer optimisation problem and a genetic algorithm is developed to tackle with it. The system dynamics and the optimisation problem solutions are presented through numerical simulations.

In Chapter 3, games of social distancing during the outspread of an epidemic are studied. These games take place among agents whose interactions have a networked structure i.e., each agent communicates only with her neighbors and these local interactions affect the spread of the epidemic. The difference between these games is the information available to the agents. Two cases are studied, one that the agents have perfect local information for the health states of their neighbors and one that the possess only statistical information for the prevalence of the epidemic. In both cases, the Nash equilibria are computed through proper decentralised algorithms. Comparative studies for the two games and various case studies are presented through simulations.

Finally, in Chapter 4, a stochastic min-max consensus protocol is introduced and analyzed. Despite the fact that it is not a game theoretic application, consensus is definitely a core problem in the field of networked systems and consensus protocols with enhanced characteristics are a thriving research area with many industrial applications. The protocol presented in Chapter 4 is proven to converge almost-surely in finite time and it could be suitable for security applications, where the agents need to converge to some common, yet random, state that could not be eavesdropped. Moreover, this protocol is implemented using only samples of the agents' states saving both bandwidth and power for communication, which are usually scarce resources in multiagent systems.

Chapter 2

Manipulative Behaviors in a Social Choice Procedure and Network Topology Design to Affect their Effects

2.1 Introduction

In recent years great progress has been made in the mathematical modeling and study of social phenomena. A topic of current interest is the study of the evolution of social agents' opinions about a certain issue. The knowledge of the mechanisms of the formation and the propagation of the agents' opinions are very useful in several fields. For example, in marketing the advertisers care about the opinion of the consumers for the advertised product and in politics the politicians care about the opinion of the agents about their agenda. Thus, a lot of work has been done in this field [31],[43],[44],[111],[57],[58],[42],[75],[4],[49],[48],[1],[41], many interesting cases have been modeled and analysed, some of which are summarized in [99], [45], and new ideas continue to be proposed and studied up to now [37],[39],[38].

Many of these works e.g., [31], [48], consider a single state for the agents, modelling their opinion, belief or attitude about an issue, and they study the dynamics of this state. The dominant mechanism that determines the evolution of the opinions is considered to be the averaging of the opinions of the agent's peers. The reason for this modelling are the tendencies of an agent to imitate her peers and to conform to her social group attitudes, which are both well-studied social phenomena. In fact, this modelling of opinion dynamics has been verified to be realistic by experimental data of a field research in India [20].

However, in many cases, such as social choice procedures (e.g. elections, referendums, polls), the organisation who studies the opinion dynamics cares to predict or to affect the outcome of this procedure, which is determined by the agents actions or behaviors. So, the question whether an agent's opinion imply a specific behavior-action naturally arises. The answer that the field of social psychology gives to this question is negative, in many cases the opinions do not imply specific actions [77],[124],[50]. Behavior is not solely dependent on one's beliefs but is drastically affected by the situations and in some cases behavior affects ones attitudes and beliefs [84].

In addition to moral and situational factors, game theory suggests that an agent's behavior is also dependent on her desire to maximize her private interests [117]. So, the action-behavior of an agent is also shaped by her utility gained form the outcome of the social choice procedure, which also depends on the other agents' actions. This indicates that an agent's action depends on her neighbors' actions and it is a best response to them. This perspective adds the useful insight that the agents usually act antagonistically to their neighbors and they do not just conform to their peers' pressure [74].

An advantage of the game theoretic modelling for the agents' actions is that it can explain better the emergence of manipulative behaviors in social choice procedures, which is a topic of significant interest. Several recent studies on several countries like U.S. [13],[6] and Argentina [112] indicate that social networks have become an arena of manipulative behaviors [125]. Paid brokers of political parties, fake accounts (bots), echo chambers, organised disinformation (fake news, slandering) are some of the manipulation techniques that have arisen in the fertile ground of the online political conversations. Furthermore, in this new environment of political struggle each agent may act in a manipulative way in an effort to pull the social outcome to her favor, however, she may be less manipulative than an expert of the previous categories. Such behaviors are considered in some recent works [1],[37],[39], [38].

In this work, we extend a model introduced in [38] describing a social choice procedure, where the population structure is modeled by an undirected graph and the agents' actions depend both on their opinions, which evolve dynamically in our model, and on their neighbors' actions. Specifically, we consider that each agent has an internal belief or opinion, which evolves in time in a way modelling a tendency of conformity to the public opinion. Each agent has also an expressed action in the social choice procedure. Each opinion matches to a proper action. However, the action of each agent isn't identical to her proper action, but it derives from the minimization of a criterion modeling the tendency of the agent to manipulate, i.e. to deviate from her proper action in order to pull the social outcome to her favor.

The resulting game between the agents is considered to be repeated in discrete time steps. The action shaping criteria of each agent retain the same form at each step. So, we formulate a series of one-step games where we seek for the Nash strategy profiles. These strategy profiles result in a dynamic rule for the actions of the agents coupled with the opinion dynamics. It is interesting that in the case that the agents are highly manipulative these dynamics become unstable, since the social outcome stands as a tug of war among the agents who try to pull it to their side.

Motivated form this fact, we study the stability properties of these dynamics and we deduce a sufficient condition that guarantees the convergence of the system to a bounded state. This condition implicates the manipulative tendencies of the agents and the graph structure with the stability of the system, stating that the acceptable manipulative behavior of an agent is relative to her position in the graph. Simulations are presented in order to examine how the opinion and action dynamics behave over several well known graph structures, such as random graphs, lattices and small world graphs.

Subsequently, we consider the problem of changing the social network's topology in order to influence the effects of manipulative behaviors. The network topology has been chosen as our designing parameter for two basic reasons. At first, the network topology is a parameter that the social network's administrator can affect and thus influence the agents' behaviors in an indirect way, which may lead to less effort and costs than the enforcement of strict rules to the users of the network. Secondly, the network topology design is an emerging problem in many scientific fields nowadays, such as security [72], multi-agent systems (MASs) [83],[110], communication networks [25], sensor networks [40], [65], distributed optimization [54], distributed LANs [68], [103], UAVs navigation [23], cyberphysical systems [67], convergence of mean field games [73] etc.

Special attention should be paid in two recent works, which deal with very similar problems with the one analysed in this work. The first is presented in [19], where the authors consider a random consensus protocol for discrete variables (Voter Model) and design the topology of a weighted graph (adjust the weights) to control its convergence. The main differences with our approach is that the decision variables of the topology design are real instead of integers and the constrain set is convex, thus the authors use semidefinite programming to solve this more tractable -in terms of complexity- problem. The second one [8] deals with the problem of optimal link addition to affect the outcome of an election procedure, which was introduced in [107]. The problem is very similar to ours since the authors consider strategic agents, whose decisions are affected by their neighbors actions, thus, from the topology of the network. Moreover, they develop an algorithm for the topology design, which is application oriented - as in our case - and they prove its optimality. The main difference between [8], [107] and our approach is the problem formulation since they model the elections as an ordering among the finite candidates (finite state space) and the best responses of the agents are derived by a rule of thumb, while in our approach the opinions of the agents are dynamically evolving states with real values and their actions are derived from the Nash equilibrium computation, resulting in a dynamic discrete-time system that we want to stabilize through the topology design.

A general formulation and study of a network topology design for the stabilisation of a system of unstable dynamics of interconnected agents can be applied to many problems of current interest. It must be specified that in our work the term social network corresponds to its digital realisation and not to its abstract concept of a representation of human relationships, so an administrator exists and the topology can be affected. We would like to note here that in contrast to its practicality the existence of one or more administrators in such networks raises the more intriguing question of who will control the administrators, who have the power to affect the other agents' manipulability and the final outcome.

For the topology design procedure, we study the case of an initial topology resulting in unstable dynamics and we want to find a new topology that results in stable dynamics and that is close to the initial topology with respect to the number and the exact position of their edges. This problem is formulated as an integer programming problem with a Lyapunov inequality for discrete time systems (known also as Schur's inequality) as constraint. Each decision variable of this optimisation problem represent either the existence of an edge between two agents or one of the components of the Lyapunov matrix. The constraint is nonlinear with respect to our decision variables and it can be written as a Bilinear Matrix Inequality, which is known to be a nonconvex problem in its general case [86]. A similar approach involving integer optimisation with a Bilinear Matrix Inequality constraint for the graph topology design problem has been addressed in [54], where the authors considered a LMI relaxation of the problem and a branch and bound technique to deal with the integer decision variables.

In this work, we develop a genetic algorithm to deal with this problem. This algorithm searches only for the values of the integer decision variables representing the edges of the graph, while a Linear Matrix Inequality solver is used to check the feasibility of each new topology by solving the Lyapunov inequality with the topology variables fixed, which results to be linear with respect to the symmetric matrix of the Lyapunov function. This procedure is repeated for many generations, where new topologies are produced by the application of the genetic operators.

Finally, simulations of the results of the proposed algorithm are presented. The behavior of the algorithm is studied over several different initial topologies, where the agents' parameters have been chosen properly so as to arise instabilities in the dynamics. Through the examination of these test cases, we derive conclusions on the functionality of the proposed algorithm and the relevancy of our results with the expected ones from our empirical perception of social networks and social choice procedures.

2.2 Problem formulation

2.2.1 Notation

We consider an undirected graph $G = (V, E)$. By n we denote the number of the vertices of the graph, which represent the agents. We denote by N_i the neighborhood of the agent i, $N_i = \{j : (i, j) \in E\}$ and by d_i the degree of node i, that is the size of its neighborhood. Let A be the adjacency matrix of the graph, it is a $n \times n$ symmetric matrix and its (i, j) entry is 1 if nodes i and j are adjacent to each other and 0 otherwise. Let $D = \text{diag}\{d_i\}$ be the diagonal degree matrix, $C = diag\{c_i\}$ be a diagonal matrix of the self-confidence parameters c_i and $G = \text{diag}\{g_i\}$ be a diagonal matrix of the manipulability parameters g_i. The symbol $\mathbb{1}$ stands for the $n \times 1$ vector with all its coordinates equal to 1. The symbol $\mathcal{I}$ stands for the identity $n \times n$ matrix and the symbols $e_i, i = 1...n$ stand for the standard basis of $\mathbb{R}^n$. For a set S we denote $\mathcal{X}_S$ its indicator function, i.e. $\mathcal{X}_S(x) = 1$ if $x \in S$ and $\mathcal{X}_S(x) = 0$ elsewhere. The symbolism $\lceil \cdot \rceil$ denotes rounding to the next natural number and the symbolism $\lceil \cdot \rceil_{even}$ denotes rounding to the next even natural number. The space of the square $n \times n$ symmetric positive definite matrices is denoted $\mathcal{M}_n^{S+}$. The symbol A^T stands for the transpose of the matrix A and the symbol $\lambda_i(A)$ denotes the i-th eigenvalue of A. All the norms $\|\cdot\|$ that have no subscript stand for the 2-norm.

2.2.2 Derivation of the Opinion Dynamics

At first, the mechanism that determines the evolution of the agents' opinions is studied. The opinions, beliefs or attitudes of the agents are a state variable, that expresses what they believe about an issue and not what they actually do. The opinion/attitude of the agent i is denoted by $\theta_i(k)$ at each time step k, and its value is a real number. In field researches, attitudes are usually measured in a five point scale, however, we consider here a continuous and unbounded analogue, which is common in the opinion dynamics literature.

It is considered that the main factors that shape the opinions in time are imitation and conformity. That is, the agents' opinions tend to be affected with their neighbors' opinions through continuous dialogue and finally reach a consensus. This model of opinions' evolution is well known and studied for many years [31],[44], [48]. In fact, in [44], [48] the model has been enriched with the inclusion of stubborn agents, i.e. people who insist on their initial beliefs, but since their presence affects primarily the equilibrium of the opinion dynamics and not their stability properties, we shall not include such agents in our model. So, every agent has an initial opinion $\theta_i(0)$ and she changes her opinion at each time step according to following dynamic rule:

$$\theta_i(k+1) = \frac{c_i}{d_i + c_i}\theta_i(k) + \frac{1}{d_i + c_i}\sum_{j \in N_i} \theta_j(k) \tag{2.1}$$

where c_i is a factor analogue to the self-confidence of the agent for her opinion.

2.2.3 Derivation of the Action Dynamics

The actions of the agents represent what they actually do, in our case what they choose in the social choice procedure. The action/behavior of each agent is denoted by $u_i(k)$ at each time step k and its value is a real number. As with the opinions, the actions could also be modeled to take values in a discrete scale, however, in this work we consider a continuous relaxation of that more difficult problem.

In contrast with the opinions which are shaping by a progressive conformity to the average beliefs, the criteria determining the action of each agent in every time step depict the tendency of the agents to manipulate the social outcome to their favor. That is, each agent may deviate her action from the one dictated by her beliefs in order to pull the social outcome towards her desired direction. In other words, as pointed out in [39], it is a common phenomenon in politics that the people who disagree with what they perceive as the expected social outcome tend to overstate their opinions, leading their neighbors to misperceptions of the public opinion and conform to these false estimations, thus pulling the social outcome to their favor. For this reason, an important parameter of their criteria is their estimation of the social outcome, based on their available information.

Assumption 1. *It is assumed that the agents have local information of the other agents' actions, that is they know only the actions of their neighbors.*

Assumption 2. *It is assumed that the information pattern is Markovian, i.e. at each time step they know only the last actions of their neighbors forgetting the past.*

So, the available information for each agent is:

$$I_i(k) = \{\theta_i(k), \theta_j(k), u_j(k-1), \forall j \in N_i\} \tag{2.2}$$

According to this information pattern the estimated social outcome for each agent is her local average, evaluated on the available samples at time k:

$$\tilde{u}_i(k) = \frac{\sum_{j \in N_i} u_j(k-1) + u_i(k)}{d_i + 1} \tag{2.3}$$

Based on the aforementioned concepts the criteria that determine the actions of each agent are dependent on her current opinion and on her locally estimated social outcome, so they are defined at each time step as follows:

$$J_i^k(I_i(k)) = (u_i - \phi_i(\theta_i))^2 + g_i(\tilde{u}_i - \phi_i(\theta_i))^2 \tag{2.4}$$

where $\phi_i(.)$ is a continuous transformation matching each agent's opinion to a desired behavior-action. The first term of the cost function $(u_i(k) - \phi_i(\theta_i(k)))^2$ indicates the self-consistency of the agent, i.e. how close her action is to an action consistent with her opinion, while the second term $g_i(\tilde{u}_i - \phi_i(\theta_i(k)))^2$ indicates the manipulative/opportunistic ends of the agent, i.e. how much she cares to affect the social outcome through her action so as to bring it close to her desirable outcome. The parameters g_i determine the ratio between self-consistent and manipulative behaviour for each agent.

Remark 1. *If Assumption 2 is relaxed by adding memory to the agents, so as to be able to predict the social outcome based on all the previous actions of their neighbors, the one-step Nash game examined here will be converted to a dynamic one. The dynamic game is of high complexity, so assuming that the social agents have bounded rationality and they do not seek to solve a difficult problem to determine their social behavior, we deal with the one-step Nash game which is tractable.*

Assuming that the agents choose their actions rationally based on their criteria we seek for the Nash equilibrium solution of the one step game. These best-response actions derive from the solution of the following system of equations:

$$\{\frac{\partial J_i^k}{\partial u_i} = 0\} \tag{2.5}$$

which have the following form:

$$\frac{\partial J_i^k}{\partial u_i} = 0 \Rightarrow 2(u_i - \phi(\theta_i)) + 2g_i\left(\frac{\sum_{j \in N_i} u_j + u_i}{d_i + 1} - \phi_i(\theta_i)\right)\frac{1}{d_i + 1} = 0$$

solving these equations with respect to u_i and using the information pattern $I_i(k)$ to evaluate each quantity in accordance with the available sample at time k we obtain the following dynamics for the actions:

$$u_i(k+1) = \left(1+\frac{d_i g_i}{g_i+(d_i+1)^2}\right)\phi_i(\theta_i(k+1)) - \frac{g_i}{g_i+(d_i+1)^2}\sum_{j\in N_i} u_j(k) \tag{2.6}$$

Introducing the following notation:

$$G_\theta = \mathrm{diag}\left\{1+\frac{d_i g_i}{g_i+(d_i+1)^2}\right\}, \tag{2.7}$$

$$G_u = \mathrm{diag}\left\{\frac{g_i}{g_i+(d_i+1)^2}\right\} \tag{2.8}$$

and

$$A_u = G_u A \tag{2.9}$$

we rewrite the equation (2.6) in matrix form:

$$u(k+1) = G_\theta \Phi(\theta(k+1)) - A_u u(k) \tag{2.10}$$

where $u(k) = [u_1(k)...u_n(k)]^T$ and $\Phi(\theta(k+1)) = [\phi_1(\theta_1(k+1))...\phi_n(\theta_n(k+1))]^T$.

2.3 Stability Analysis

2.3.1 Known results on opinion dynamics

For the evolution of the opinions of the agents (2.1), which can be summarized using the matrix notation

$$A_\theta = (D+C)^{-1}(A+C) \tag{2.11}$$

to the following expression:

$$\theta(k+1) = A_\theta \theta(k) \tag{2.12}$$

where $\theta(k) = [\theta_1(k),...,\theta_n(k)]^T$ and A_θ is a row-stochastic, aperiodic matrix. So, $\theta(k)$ converges to a limit θ^c which is actually a consensus on each connected subgraph. For some results on these the reader could study [31] and for a more general description one could study the criteria summarised in [95]. So the following statements hold:

$$\|\theta(k)-\theta^c\| \to 0 \tag{2.13}$$

2.3.2 Stability analysis of the coupled opinion and action dynamics

We continue our analysis by considering the augmented state vector

$$z(k) = [\theta_1(k), ..., \theta_n(k), u_1(k), ..., u_n(k)]^T$$

and the resulting augmented system dynamics. For simplicity of the presentation we will use the notation $\Phi \circ A_\theta \theta(k)$ to denote the nonlinear function $\Phi(\theta(k+1))$. So we obtain the following dynamics:

$$z(k+1) = \begin{bmatrix} A_\theta & 0 \\ G_\theta \Phi \circ A_\theta & -A_u \end{bmatrix} z(k) \tag{2.14}$$

Lemma 1. *If there exists a symmetric, positive definite matrix P such that $A_u^T P A_u - P < 0$ and the function Φ is continuous in $\mathcal{R}^n$ and locally Lipschitz in a neighborhood of θ^c with a Lipschitz constant L_Φ, then the coupled dynamics* (2.14) *have an equilibrium which is globally asymptotically stable.*

Proof. At first, we define the P-norm of a vector x: $\|x\|_P := \sqrt{x^T P x}$ and of a matrix M: $\|M\|_P := \sup_{\{\|x\|_P=1\}}\{\|Mx\|_P\}$. So, we have that if $A_u^T P A_u - P < 0$ holds then $\|A_u\|_P < 1$.

The opinion dynamics, $\theta(k+1) = A_\theta \theta(k)$, it is known to be stable as we have already discussed. So, $\exists K : \forall k > K$ $\theta(k)$ belongs to a neighborhood of θ^c where the mapping Φ is Lipschitz. Thus $\forall k > K$ the following holds for the actions:

$$\begin{aligned} \|u(k+1)-u(k)\|_P =& \|G_\theta \Phi(\theta(k+1)) - A_u u(k) - G_\theta \Phi(\theta(k)) + A_u u(k-1)\|_P \\ \leq& \|G_\theta \Phi(\theta(k+1)) - G_\theta \Phi(\theta(k))\|_P + \|A_u u(k) - A_u u(k-1)\|_P \\ \leq& L_\Phi \|G_\theta\|_P \|\theta(k+1)-\theta(k)\|_P + \|A_u\|_P \|u(k)-u(k-1)\|_P \end{aligned} \tag{2.15}$$

let $a = \|A_u\|_P < 1$ and

$$\delta_k = L_\Phi \|G_\theta\|_P \|\theta(k+1)-\theta(k)\|_P \to 0$$

due to (2.13) and the fact that $\|x\|_P \leq \sqrt{\lambda_{max}(P)}\|x\|$.Thus, denoting $x_k = \|u(k) - u(k-1)\|_P$, we rewrite the previous inequality:

$$x_{k+1} \leq ax_k + \delta_k \tag{2.16}$$

with $a < 1$ and $\frac{\delta_k}{1-a} \to 0$. Inequality (2.16) satisfies the conditions of lemma 3, p.45 of [97] and consequently it converges to zero, thus the sequence $\|u(k+1) - u(k)\|_P$ is convergent to zero, so the sequence $u(k)$ is convergent to an equilibrium point. So finally, the coupled dynamics have an equilibrium which is globally asymptotically stable. □

Remark 2. *These results can be generalised in the cases of directed, weighted graphs with switching topology, modeled by an adjacency matrix $A(k) = [a_{ij}(k)]$ at each time step k, where $a_{ij} > 0$ if node i receives information from node j, if the following two conditions hold:*

1. *There exists $T \geq 0$ such that for every interval $[k, k+T]$ the union of the interaction graph across the interval contains a spanning tree.*
2. *There exists a symmetric, positive definite matrix P such that $A_u^T(k)PA_u(k) - P < 0$ for all $k = 1...\infty$.*

That generalisation can be derived from Proposition 1 in [91] or Lemma 1 in [70]. The existence of a spanning tree can be characterised as a rational assumption for networks modelling social relationships. We argue for this because social relationships are usually mutual, yet not of the same intensity for both parts, so they can be modelled by a weighted, directed and strongly connected graph. If a directed graph is strongly connected then it has a spanning tree.

The usefulness of Lemma 1 arises form the fact that the opinion dynamics are stable for every graph structure as the matrix $A_\theta = (D+C)^{-1}(A+C)$ has the desired properties for every adjacency matrix A and its degree matrix D. So, this lemma enables us to focus on the stabilization of the action dynamics, through the graph design and the consequent tuning of the matrix A_u, guaranteeing that the coupled dynamics will remain stable for every such design.

From the previous lemma, using $P = \mathcal{I}$ in the Lyapunov matrix inequality (thus $\|\cdot\|_P = \|\cdot\|_2$) and $G_u A = G_u(D+\mathcal{I})(D+\mathcal{I})^{-1}A$ we can derive the following simple but restrictive stability condition for the spectral radius of $G_u(D+\mathcal{I})$, $\rho(G_u(D+\mathcal{I})) = max\{|\lambda_i(G_u(D+\mathcal{I}))|, i = 1...N\}$ to be less than one as well or equivalently:

$$\frac{(d_i+1)g_i}{g_i + (d_i+1)^2} \leq 1 \Rightarrow g_i \leq d_i + 2, \forall i \tag{2.17}$$

since it this case

$$\|A_u\| \leq \|G_u(D+\mathcal{I})\|\|(D+\mathcal{I})^{-1}A\| \leq max\{|\lambda_i(G_u(D+\mathcal{I}))|, i=1...N\}\|(D+\mathcal{I})^{-1}A\|$$

because the matrix $G_u(D+\mathcal{I})$ is diagonal. For the second norm it holds:

$$\|(D+\mathcal{I})^{-1}A\| \leq \|(D+\mathcal{I})^{-1}\|\|A\| = \frac{1}{d_{max}+1}\|A\|$$

and for $\|A\|$ it holds

$$\|A\| \leq \sqrt{\|A\|_\infty \|A\|_1} = d_{max}.$$

So,

$$\|(D+\mathcal{I})^{-1}A\| \leq \frac{d_{max}}{d_{max}+1} < 1.$$

Remark 3. *We state this simple observation here because we can exploit its simplicity to use it as a heuristic for a stable topology design. That is, since this condition guarantees that the coupled dynamics converge on a graph with* $min\{d_i\} \geq max\{g_i\} - 2$ *we know that a ring lattice of degree* $d_1 = \lceil max\{g_i\} - 2 \rceil_{even}$ *is a topology that stabilizes these dynamics.*

2.3.3 Simulations on the model's stability properties

We present here some simulations of the aforementioned dynamics over different graph structures, that motivated us to formulate the topology design problem. In these simulations we consider a network of $n = 20$ agents participating in a repeated social choice procedure for $T = 100$ times. The parameters c_i indicating the obstinateness of the agents are chosen from the interval $[10, 100]$. The parameters g_i indicating the manipulative tendencies of the agents are randomly chosen from the interval $[0, 15]$. Their initial opinions are randomly chosen from the $[0, 10]$ interval. Their initial actions are the desired ones according to their initial opinions $u_i(0) = \phi(\theta_i(0))$, where the function Φ is considered to be $\Phi(\theta) = 10\tanh(\theta/10)$, which is both continuous and locally Lipschitz.

Firstly, we present the convergent opinion and action dynamics (Figure 2.2) on a realization of a random graph [36] with edge probability $p = 0.4$ (Figure 2.1). In the presented case the graph has $|E| = 81$ edges and the spectral radius of the resulting matrix A_u equals $\lambda_{max}\{A_u\} = 0.7774$, so it has the necessary stability properties.

Subsequently, a case of nonconvergent dynamics will be presented. The dynamics (Figure 2.4) result from a realization of a random graph with edge probability $p = 0.3$ (Figure 2.3), which has $|E| = 54$ edges and $\lambda_{max}\{A_u\} = 1.0418$.

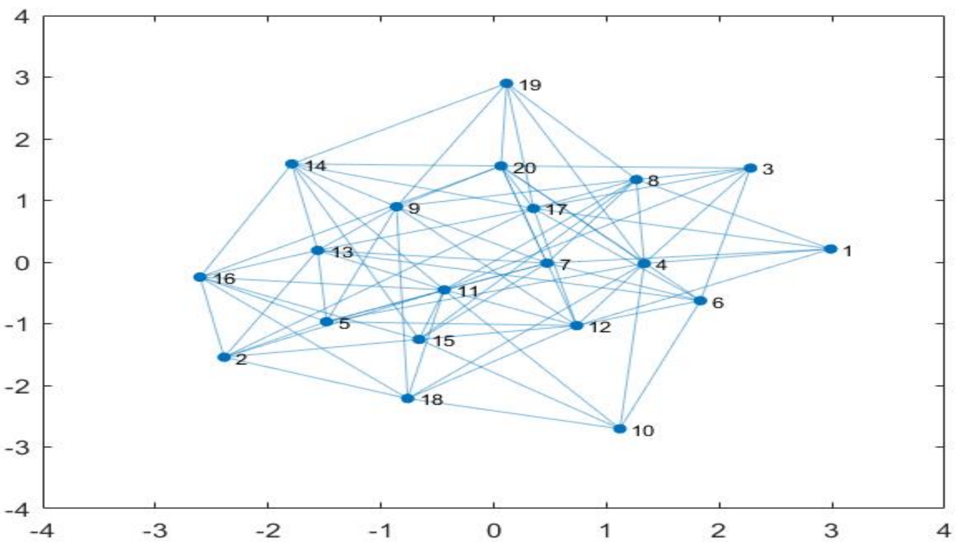

Figure 2.1 A random graph with edge probability p=0.4.

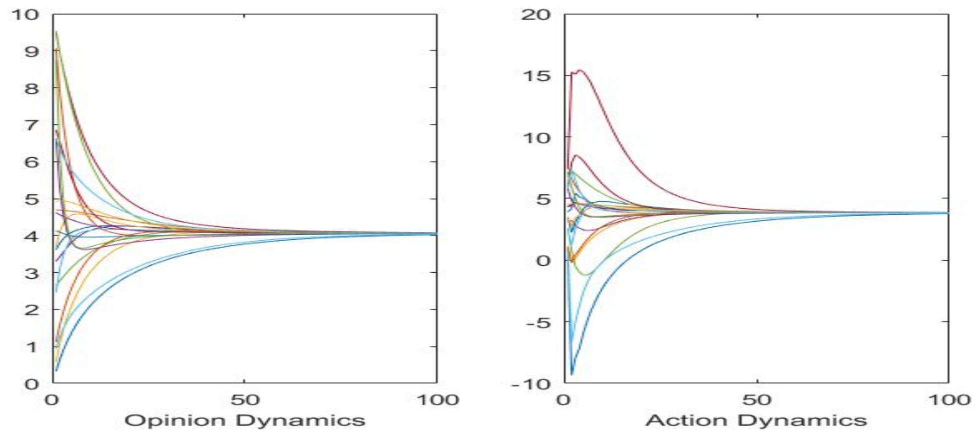

Figure 2.2 Opinion and action dynamics.

We consider now the problem of choosing a proper graph structure, which will result in stable dynamics and it will be as close as possible to the aforementioned unstable structure with respect to the edge number $|E|$ in this case. We make several experiments beginning from an L^*-lattice (a graph where all the agents have the same degree L^*), which satisfies our sufficient condition ($L^* > g_{max} - 2$), $L^* = 14$ in this example. Then we relax this condition by considering lattices of smaller node degree until the dynamics become unstable, as shown in Table 2.1.

The most interesting observation we made from our experiments was that while the 6 degree lattice results in unstable dynamics if we rewire some of its edges and thus create a small

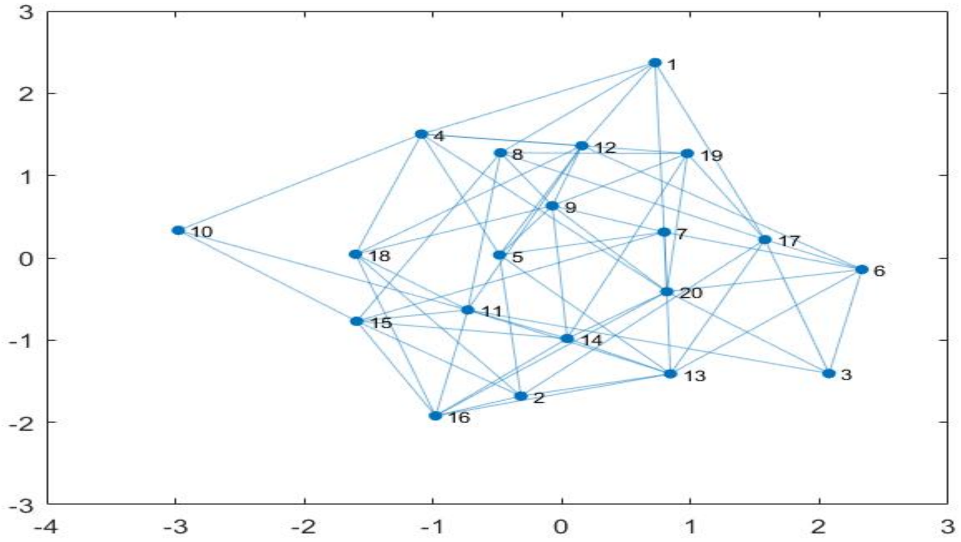

Figure 2.3 A random graph with edge probability p=0.3.

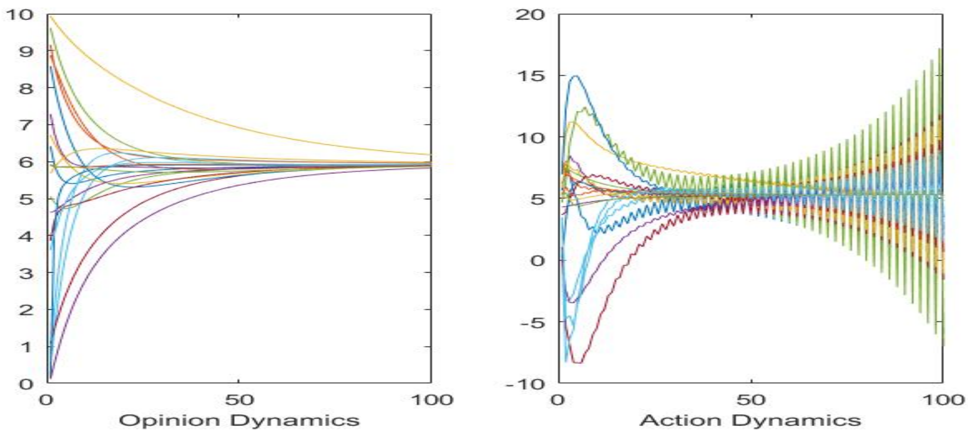

Figure 2.4 Opinion and unstable action dynamics.

Graph structure	$\lambda_{max}(A_u)$	$\|E\|$
L^*-lattice	0.4042	140
8-lattice	0.7758	80
6-lattice	1.0114	60
Small-world	0.9491	60

Table 2.1 Stability of several graph structures

world graph, as introduced by J. Watts and S.Strogatz (1998), the dynamics become stable. This indicates that a well structured topology -whose properties can be studied analytically-is not necessarily the best choice for our problem, on the opposite the introduction of some random rewirings results in better structures. This was a motivation for the following general formulation of the topology design problem, which is not restricted on several special classes of topologies.

2.4 Network topology design for the stabilization of the action dynamics

2.4.1 Notation and Problem statement

The network topology design problems are emerging in many different fields [72]-[73] and in more formulations they are considered to be difficult (NP-hard) problems. That is because the decision variables stand for the presence, the addition or the removal of nodes or links and so they take integer values, resulting in Integer Programs with various types of constraints.

Similarly, in our case we consider the vector

$$x \in \{0,1\}^{n(n-1)/2},$$

which denotes the occurrence of a change of an edge -addition or removal of an edge- in the existing graph structure and constitutes our decision variables. The nodes of the graph remain unchanged.

Let $\{P^k, k = 1 \ldots \frac{n(n+1)}{2}\}$ be a basis of the symmetric $n \times n$ matrices. Specifically, consider the matrices P^k with $P^k_{ij} = P^k_{ji} = 1$ if $i = \max_{m \geq 0}\{\sum_{l=1}^{m-1}(n-(l-1)) \leq k\}$ and $j = i - 1 + k - \sum_{l=1}^{i-1}(n-(l-1))$ and $P^k_{ij} = 0$ elsewhere. The diagonal matrices of this basis, i.e. $\{P^k : k \in K_d = \{\sum_{l=1}^{i-1}(n-(l-1)) + 1, i = 1 \ldots n\}\}$, we will denote them P^i_d since each $k \in K_d$ corresponds to an $i \in \{1 \ldots n\}$.

Example 1. *We present for example the aforementioned basis for the* 2×2 *symmetric matrices:*

$$P^1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad P^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad P^3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

The set $K_d = \{P^1, P^3\}$*, so* $P^1_d = P^1$ *and* $P^2_d = P^3$*.*

Using this notation we can write

$$A_0 = \sum_{k \notin K_d} x_0(k) P^k,$$

where the vector x_0 stands for the coordinates of A_0 with respect to the aforementioned basis $\{P^k, k=1...\frac{n(n+1)}{2}\}$ except its diagonal elements whose coordinates are all zero. From the definition of P^k it holds that $x_0(k) \in \{0,1\}$.

The topology design procedure consists of the addition of some new edges and the removal of some existing edges. So, we define the following sign function $\mathcal{S}_{x_0}(k)=1$ if $x_0(k)=0$ and $\mathcal{S}_{x_0}(k)=-1$ if $x_0(k)=1$, which multiplied with the vector of changes x indicates which changes correspond to an addition of an edge and which to a removal.

So the adjacency matrix of the graph depends linearly on the changes' vector x:

$$A(x) = A_0 + \sum_{k=1}^{n(n-1)/2} x(k)P^k\mathcal{S}_{x_0}(k) \tag{2.18}$$

form this equation we deduce that $A(x) = [\mathcal{L}^A_{ij}(x)]$ where $\mathcal{L}^A_{ij}(x)$ are linear functions of x. The degree matrix changes accordingly:

$$D(x) = \sum_{i=1}^{n} e_i(A(x)\mathbb{1})^T P_d^i \tag{2.19}$$

which also depends linearly on x, i.e. $D(x) = \mathrm{diag}\{\mathcal{L}^D_i(x)\}$ where $\mathcal{L}^D_i(x)$ are linear functions of x.

Subsequently, we define the matrix functions:

$$\begin{aligned} G_u(x) &= G(G+(D(x)+I)^2)^{-1} \\ &= \mathrm{diag}\{\frac{g_i}{g_i+(\mathcal{L}^D_i(x)+1)^2}\} \end{aligned} \tag{2.20}$$

and

$$A_u(x) = G_u(x)A(x) \tag{2.21}$$

which are nonlinear with respect to the decision variables x.

Applying the Lyapunov stability criterion on the matrix $A_u(x) = G_u(x)A(x)$ we obtain the following matrix inequality for $P > 0$ and x:

$$A(x)G_u(x)PG_u(x)A(x) - P \leq Q \tag{2.22}$$

The matrix Q is a negative definite matrix, for example $Q = -q\mathcal{I}$, where q is a design parameter affecting the stability properties of the system as well as the size of the feasible region of the optimisation problem. In the simulations presented in the next section this parameter takes values of the order: $q \sim 10^{-2}$.

Let the $F_x = \{x : \exists P > 0 : A(x)G_u(x)PG_u(x)A(x) - P \leq -q\mathcal{I}\}$. This set contains all the feasible designs, i.e. the vectors x for witch the induced graph described by the adjacency matrix $A(x)$ has the desired stability properties.

In order to choose an element of the aforementioned feasible set as a best design, we consider the criterion of the minimum change from the initial graph structure, which is a natural criterion as especially on graphs representing social interactions it may be very difficult to persuade someone to abandon a friend or make a new one. So we consider the minimization of $\|x\|_1$, which is equivalent to the minimization of the linear objective $\mathbf{1}^T x$. The resulting problem is:

$$\min_{x,P}\{\mathbf{1}^T x\} \tag{2.23}$$

$$x \in \{0,1\}^{n(n-1)/2} \tag{2.24}$$

$$\exists P > 0 : A(x)G_u(x)PG_u(x)A(x) - P \leq -q\mathcal{I} \tag{2.25}$$

Remark 4. *If for some reasons some edges of the network are considered to be more important than others, or the cost to add or remove them is different, we can formulate a similar optimisation problem substituting the objective by a weighted sum of the changes $w^T x$, $w_i \geq 0$. Moreover, several linear constraints may be added so as to describe restrictions on the design parameters due to special structural characteristics of the network, which may be important to be preserved or due to special characteristics of several nodes, whose neighborhood cannot be affected. These changes in the optimisation problem formulation do not increase the difficulty of the problem as it lies on the constraint* (2.25).

In order to simplify the nonlinear, non-polynomial (on the decision variables x) constraint $\exists P > 0$: $A(x)G_u(x)PG_u(x)A(x) - P \leq -q\mathcal{I}$ we consider the change of variables $Z = G_u(x)PG_u(x)$ and prove the following proposition.

Proposition 1. *For every point x, if there exists a matrix $Z > 0$:*

$$A(x)ZA(x) - G_u^{-1}(x)ZG_u^{-1}(x) \leq -q\mathcal{I} \tag{2.26}$$

then there exists a matrix $P > 0$:

$$A(x)G_u(x)PG_u(x)A(x) - P \leq -q\mathcal{I}. \tag{2.27}$$

Proof. We use the mapping $Z = G_u(x)PG_u(x)$ from $P \in \mathcal{M}_n^{S+}$ to $Z \in \mathcal{M}_n^{S+}$. For each element of the matrices Z and P it holds that :

$$z_{ij} = \frac{g_i g_j}{[g_i + (\mathcal{L}_i^D(x)+1)^2][g_j + (\mathcal{L}_j^D(x)+1)^2]} p_{ij},$$

which is a bijection. Moreover, if $Z > 0$ then for $P = G_u^{-1} Z G_u^{-1}$ it holds that for every vector x:

$$x^T P x = x^T G_u^{-1} Z G_u^{-1} x = v^T Z v > 0$$

for $v = G_u^{-1} x$, so $P > 0$. Finally, substituting the change of variables $Z = G_u(x)PG_u(x)$ in

$$A(x)ZA(x) - G_u^{-1}(x) Z G_u^{-1}(x) \leq -q\mathcal{I}$$

we take the desired inequality

$$A(x)G_u(x)PG_u(x)A(x) - P \leq -q\mathcal{I}$$

. □

The new constraint (2.26) is polynomial in the decision variables x, so with a proper change of variables it can be transformed to a Bilinear Matrix Inequality (BMI). We give the following simple example, from [116] p.372, to explain this change of variables:

Example 2. *Let the polynomial inequality $x^3 + yz < 1$. Defining $w = x^2$ and $v = x$ we have the following equivalent system of bilinear inequalities:*

$$\begin{aligned} 1 - xw - yz &> 0 \\ w - xv &\geq 0 \\ xv - w &\geq 0 \\ x - v &\geq 0 \\ v - x &\geq 0 \end{aligned}$$

In our case, each element h_{ij} of the polynomial matrix $A(x)ZA(x) - G_u^{-1}(x)ZG_u^{-1}(x)$ is a 4^{th} degree polynomial of the decision variables x:

$$h_{ij} = \sum_{l=1}^{n} (\sum_{k=1}^{n} \mathcal{L}_{ik}^A(x) z_k l) \mathcal{L}_{lj}^A(x) - \frac{z_{ij}}{g_i g_j)} [g_i + (\mathcal{L}_i^D(x)+1)^2)][g_j + (\mathcal{L}_j^D(x)+1)^2)]$$

Using the fact that $(x(k))^n = x(k)$ for every n since $x(k)$ is 0 or 1 and introducing some extra variables $y_{kl} = x(k)x(l)$ we can write the polynomial matrix inequality (2.26) as a Bilinear Matrix Inequality, with the aid of the matrices of the basis $\{P^k\}$.

The feasibility of a BMI is known to be a nonconvex problem in its general case [86], so the same holds for our initial problem (2.23)-(2.25). The difficulty to deal with the BMI integer constrained problem is also discussed in [54]. Moreover, due to the difficulty of the topology design problem in general, it has to be stated here that our references in this topic [72],[73] use heuristics or meta-heuristics, except the ones considering simplifying assumptions or relaxations to deal with a convex problem in the end.

2.4.2 A genetic algorithm for the topology design problem

Genetic algorithms are a well known meta-heuristic which can be applied to obtain suboptimal solutions in a variety of difficult (NP-hard) search and optimisation problems [51]. As such, it is evident that these algorithms are a useful tool for dealing with network topology design problems and they have already been applied in this field [40], [76]. Following this direction, we develop a genetic algorithm to obtain a feasible solution for the nonconvex integer programming problem (2.23)-(2.25). In order to avoid the explosion of the dimensionality which results to a very slow convergence for the genetic algorithm, we use the genetic algorithm to search only in the space of the decision variables x rather than in the whole space (x,P). However, this search may lead to several topologies which will not satisfy the constraint (2.25). To deal with this we observe that the constraint (2.25) is linear with respect to the matrix variable P, so its feasibility can be efficiently checked with the use of a projective method based algorithm for Linear Matrix Inequalities (LMIs). So, for each new topology produced by the genetic operations we check its feasibility with an LMI solver and we drop it out of the next generation if it is infeasible. The basic characteristics of this algorithm are enlisted below:

Chromosomes: Each chromosome of the genetic algorithm is a 0-1 sequence of length $\frac{n(n-1)}{2}$ representing the vector $x_0 + x \cdot \mathcal{S}_{x_0}$ for some changes' vector x. The vectors x_0, x and $\mathcal{S}_{x_0}$ are defined in the previous section, while the symbol "$\cdot$" denotes elementwise multiplication of the two vectors.

Initial population: As initial population for the genetic algorithm we consider a specific number of feasible random perturbations of the initial topology x_0. That is we produce a number of chromosomes of the form $x_0 + x \cdot \mathcal{S}_{x_0}$, which satisfy the constraint (2.25), where x are randomly derived 0-1 sequences. The feasibility check, which is described below, is applied on these chromosomes in order to verify which of them are satisfying the constraint (2.25) and reject the others from the initial population.

Fitness function: The fitness function of the genetic algorithm coincides with the objective function of the problem (2.23)-(2.25), so it has the following form

$$\text{fitness}(\text{chromosome}) = \|\text{chromosome} - x_0\|_1 = \|x_0 + x \cdot \mathcal{S}_{x_0} - x_0\|_1 = \|x\|_1$$

Selection: For the choice of a portion of the population for the breeding of the next generation we use a simple truncation selection criterion. We choose the 50% fittest part of the population in the case the size population exceeds a specific lower bound or we hold the whole population if its size is smaller than this lower bound. The reason for this is to avoid the diminishment of the population in the case that many new offsprings are rejected because they do not satisfy the constraints. The next generation of the population is initialised by the selected part of the previous population. The truncation selection has the drawback that it may lead to elitism, that is the selection of only the temporarily best chromosomes which may be far from the global optimum. Thus, the algorithm may converge to a local minimum of the optimisation problem, but the convergence speed of the algorithm if we use another selection procedure, such as fitness proportionate selection, is much slower, so we have kept this simple method for our experimental simulations. Moreover, by choosing our initial conditions relatively close to the optimum - we initialise the algorithm with perturbations of the initial infeasible topology which are adequately close to it and feasible - we enhance our chances to find the global optimum even with this selection procedure. Of course, in cases of practical interest where great accuracy is needed and with sufficient computing power available, we can easily replace this subroutine by one applying fitness proportionate selection.

Crossover: The crossover operator considered here chooses randomly two parents form the selected portion of the population and chooses also randomly a crossover point between $1 \ldots \frac{n(n-1)}{2}$ and produces two offsprings form the two possible combinations of the parent chromosomes around this point.

Mutation: The mutation operator applied to an offspring changes each of its bits with probability $p_m = \frac{2}{n(n-1)}$, resulting on an average change of one bit per chromosome.

Feasibility check: After the production of the new offsprings with the application of the genetic operators, each offspring is checked for the feasibility of the constraint (2.25). For this we use an LMI solver, which uses a projective method algorithm, to examine the existence of a matrix $P > 0$ which satisfies the LMI (2.25), where the matrices $A(x)$ and $G_u(x)$ have the fixed values corresponding to the vector x of the offspring's chromosome $x_0 + x \cdot \mathcal{S}_{x_0}$. If this LMI is found feasible the new chromosome is added to the next generation, else it is rejected.

Termination criterion: The genetic algorithm terminates after a specified number of generations N. In fact, in the following simulations we have chosen the number of generations through experimentation so as to not observe any improvement in the objective function in the final generations. The fittest chromosome of the last generation is returned as solution for our topology design problem.

Remark 5. *This algorithm can be generalised to the case of a network topology modeled by a directed graph, with an appropriate change in the basis (P^k) of the space of the adjacency matrices and the respective change in the form of the chromosomes.*

2.4.3 Simulations of the results of the genetic algorithm

In the following simulations we consider a network of $n = 20$ agents participating in a repeated social choice procedure for $T = 300$ times. The parameters c_i are chosen randomly from the interval $[10, 100]$. The parameters g_i are randomly chosen form the interval $[0, 10]$. The function Φ which maps the opinions to the desired actions is considered to be $\Phi(\theta) = 10tanh(\theta/10)$, which is both continuous and locally Lipschitz. The initial opinions $\theta_i(0)$ are randomly chosen from the interval $[0, 10]$ and the initial actions are the ones corresponding to these opinions $u_i(0) = \phi(\theta_i(0))$. All the aforementioned parameters remain the same in both simulations.

The initial graph topology is the realisation of a random graph with edge probability $p = 0.2$ shown in Figure 2.5. The resulting opinion and action dynamics are shown in Figure 2.6, where we can see that the action dynamics are unstable.

Applying the genetic algorithm presented in the previous section to the initial graph topology we obtain the graph topology presented in Figure 2.7, which differs from the initial one only on three edges. The resulting opinion and action dynamics are shown in Figure 2.8, where we can see that the action dynamics are stable over the designed graph topology.

Comments:

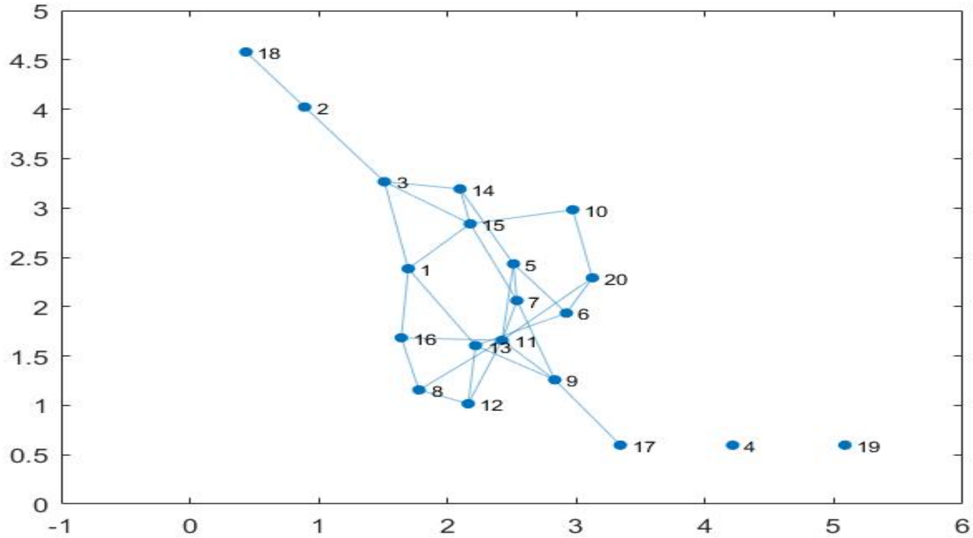

Figure 2.5 The initial graph topology, derived as a random graph with edge probability $p = 0.2$.

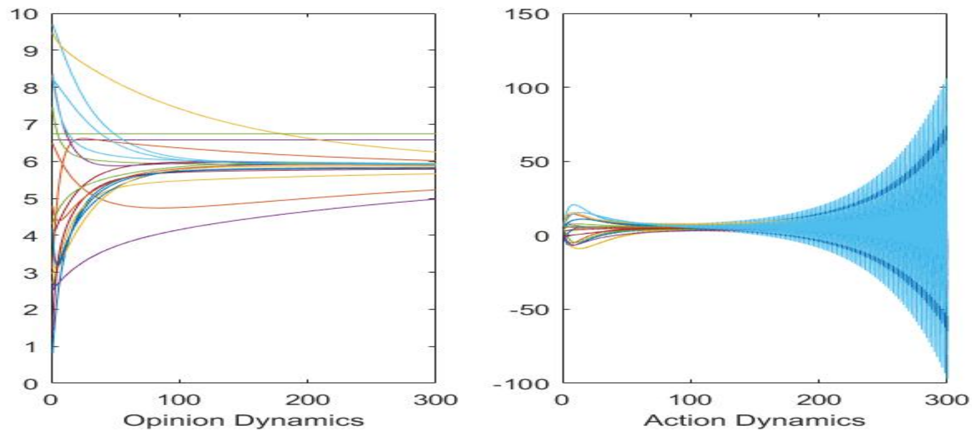

Figure 2.6 Unstable action dynamics on the initial graph topology.

1. As we observe from the simulations above the graph topology that derived from the genetic algorithm is a feasible point of our optimisation problem that satisfies the BMI constraint and it results in stable action dynamics. So, at first, our algorithm returns a feasible solution.

2. Moreover, with respect to its optimality, we note that the designed topology differs from the initial one on just 3 edges (specifically 1 edge has been removed and 2 new

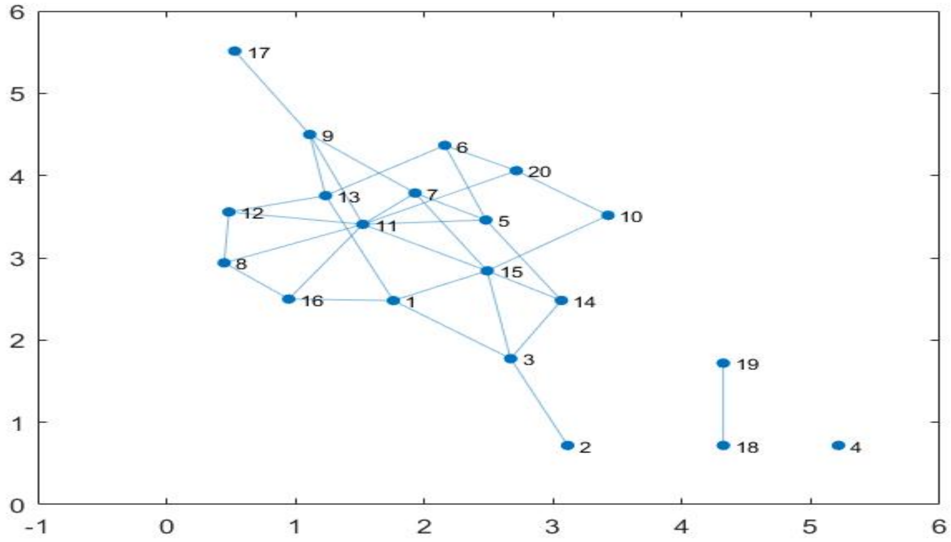

Figure 2.7 The designed graph topology by the genetic algorithm.

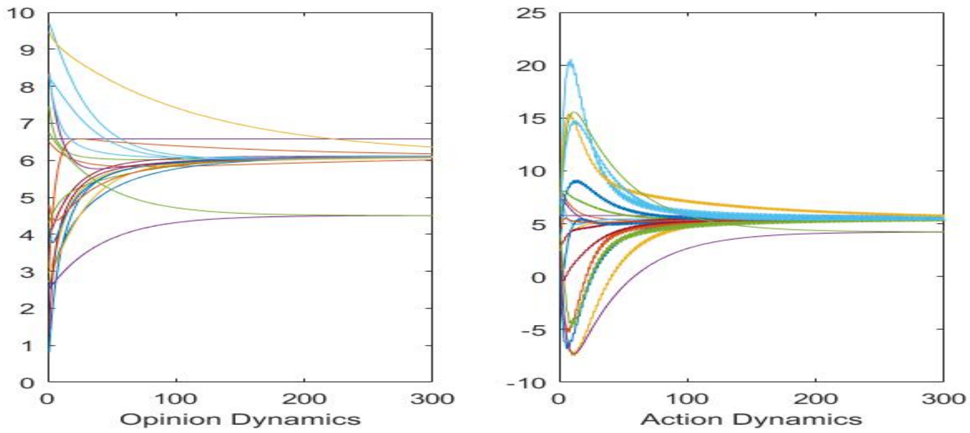

Figure 2.8 Stable action dynamics on the designed graph topology.

edges have been added), meaning that $\|x\|_1 = 3$ which is very small. It may be a suboptimal solution, but in most cases it might be an acceptable design.

3. Finally, compared with the heuristic approaches developed in section 4.3 it outperforms them by far, since the best we had achieved there was a difference of 8 on the amount, not on the exact location, of the existing edges of the two topologies, while now we achieved a difference of 3 on the exact location of the edges of the two topologies.

To the best of our knowledge there does not exist global and efficient algorithms for nonconvex integer optimisation problems, thus, the convergence of the algorithm to a feasible, yet

suboptimal, solution is a positive result per se.
Heuristic and meta-heuristic algorithms are common in the related literature [8],[107], [76, 40, 68, 72, 73]. Such algorithms are usually application oriented i.e., they are designed to tackle efficiently a specific problem. In the same way, our algorithm is designed to solve efficiently our topology design problem, based on its features, and through an appropriate design we reduce the difficult nonconvex problem to many - easier to solve - convex problems (LMI feasibility check). That design results in an efficient algorithm, especially for networks of small size, and we demonstrate its effectiveness through the simulations of many different cases in this section and in the following section.
However, we want to point also the drawbacks of our algorithm, that can be summarised in the following:

- There is no guaranteed convergence to the global optimum of the design problem and the algorithm may return suboptimal solutions.
- The algorithm can be considered slow if applied to large network topologies with limited computing power. This drawback can be tackled with distributed implementation of the algorithm in large computer centers, which are usually available to network administrators.

2.4.4 Simulations over Special Structured Initial Topologies

In the following simulations we consider a network of $n = 20$ agents and we check just the structure of the resulting topologies after the implementation of the genetic algorithm on several special structured initial topologies. The parameters g_i indicating the manipulative tendencies of the agents are chosen accordingly in each case in order to make the initial topology resulting in unstable dynamics.

Ring

For the ring topology (Figure 2.9) the parameters g_i indicating the manipulative tendencies of the agents are chosen randomly from the interval $[0, 10]$. The ring is a very sparse structure for a connected one. It has only 20 edges while 19 are needed in order to be connected. Furthermore, its stability properties are not very enhanced - even small manipulative parameters result in instabilities. So, a connected stable topology differs a lot from the initial one. That's why our algorithm returns an unconnected topology as the optimal solution, Figure 2.10. This topology has 5 edges and differs from the initial one on 15 edges. The unconnected designed topology is stable, since the isolation of the agents pauses their social interactions and results in the preservation of their initial opinions and actions, which are stable in the sense they remain bounded.

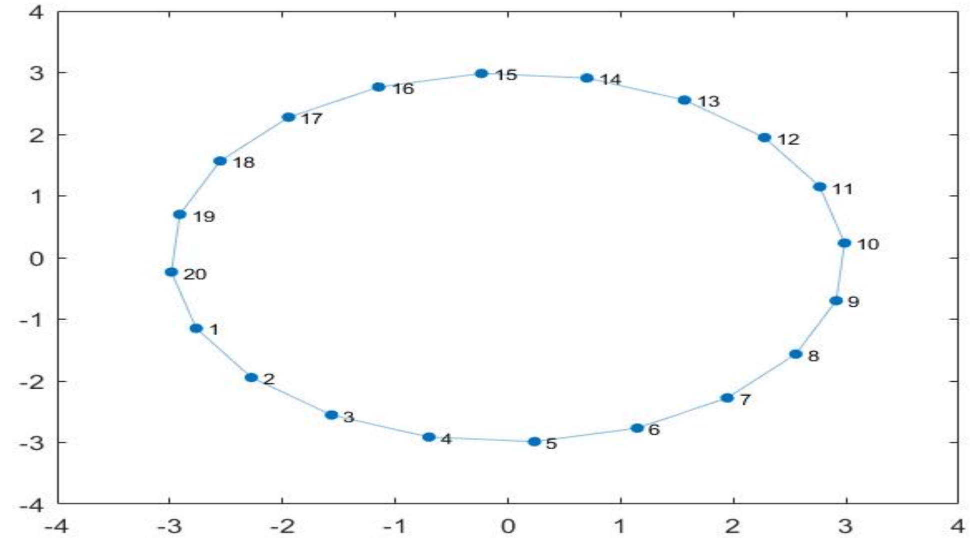

Figure 2.9 Initial ring topology

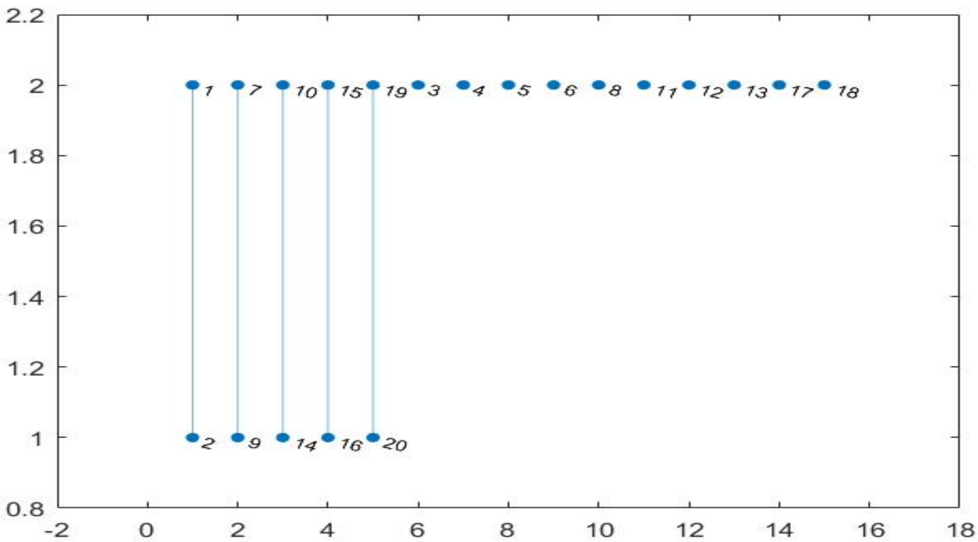

Figure 2.10 Designed unconnected topology from a ring(optimal)

Even if it is mathematically acceptable, the isolation of the agents is a bit unrealistic and in many cases undesirable design. Subsequently, we add a linear constraint in the topology design problem demanding the designed topology to have at least 19 edges -the minimum

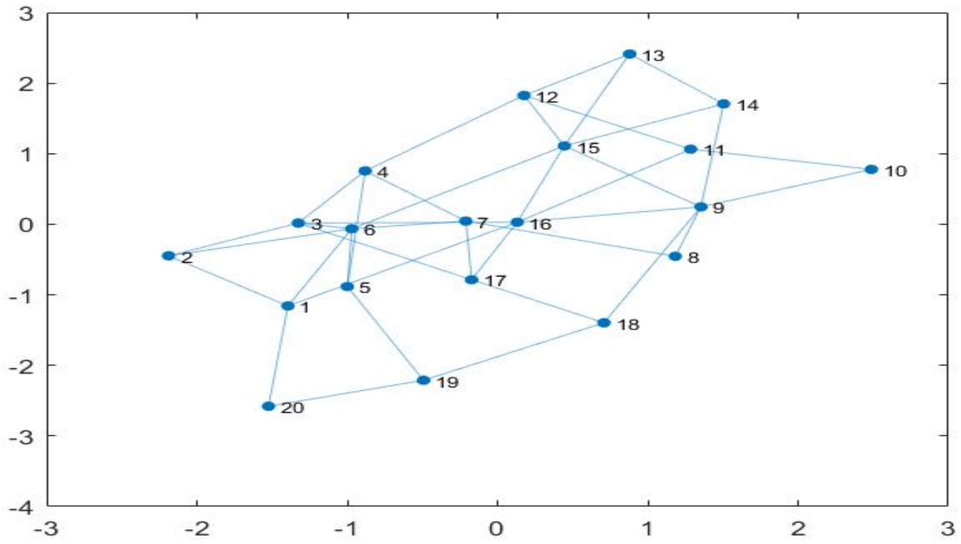

Figure 2.11 Designed connected topology from a ring

edges needed to be connected. This is a heuristic approach, since this constraint does not guarantee that the topology will be connected. However, it is a simple constraint, which interestingly works and we obtain a connected topology shown in Figure 2.11, which has 39 edges and differs from the initial one on 20 edges.

4-lattice

For the 4-lattice (Figure 2.12) the parameters g_i indicating the manipulative tendencies of the agents are chosen randomly from the interval $[0,20]$. This increase in the values of the manipulation parameters shows from the beginning that the lattices have enhanced stability properties in comparison with the ring, as it is expected since they are more dense and well connected topologies. The 4-lattice depicted in Figure 2.12 has 40 edges. Our design results in the topology of Figure 2.13 which has 43 edges and differs from the initial one on 5 edges.

Star

For the star topology (Figure 2.14) the parameters g_i indicating the manipulative tendencies of the agents are chosen randomly from the interval $[0,20]$, except the one of the central node which is chosen much larger (here $g(1) = 70$). That is because the star structure is a very robust one with respect to its stability properties, since the central node is very difficult to manipulate and to be manipulated as she has the most neighbors she could have. So, the parameters should be chosen large enough in order to arise instabilities on this initial

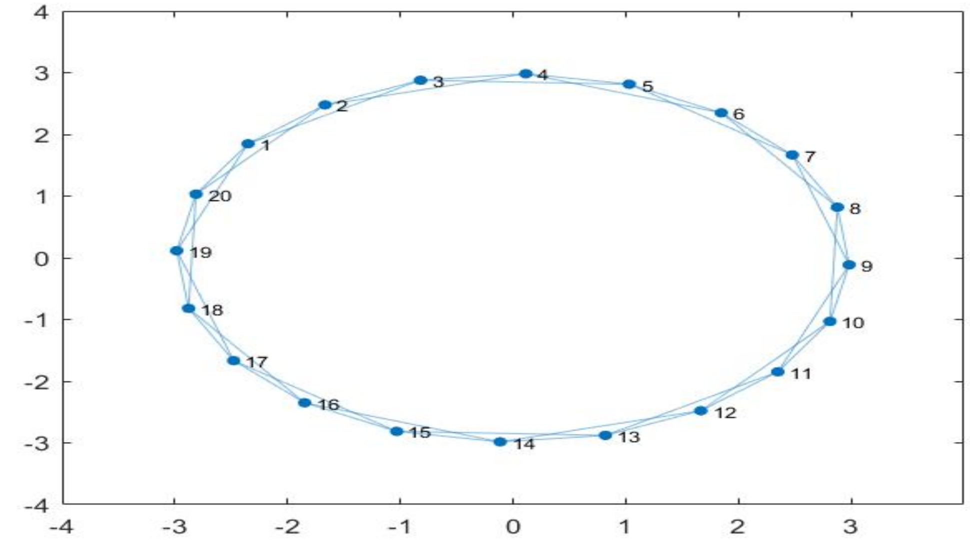

Figure 2.12 Initial 4-lattice topology

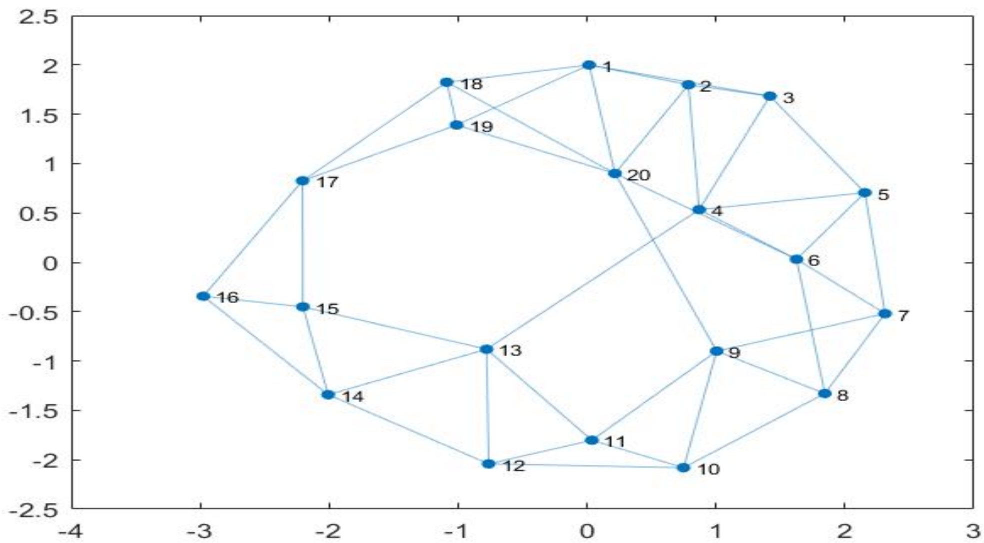

Figure 2.13 Designed topology from 4-lattice

topology. Moreover, the star graph has the least possible edges needed to be connected (19 edges), so it seems to be a very robust design for the number of its edges. That is the reason why our algorithm converges to an unconnected topology, Figure 2.15, which is closer to the star topology than any connected stable one. It has only 3 edges and it differs form the initial topology on 18 edges. It shall be noted here that, as in the case of the ring, the unconnected designed topology is stable.

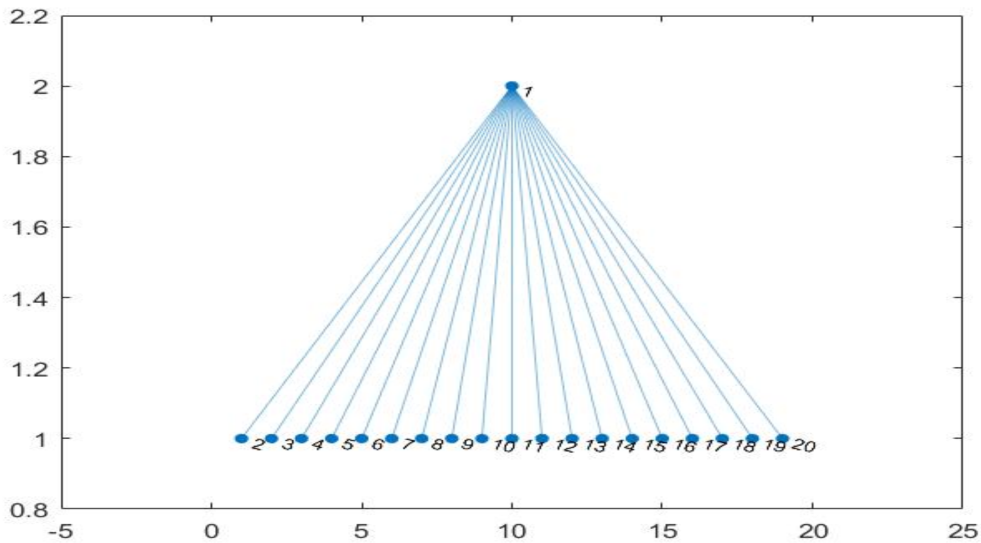

Figure 2.14 Initial star topology

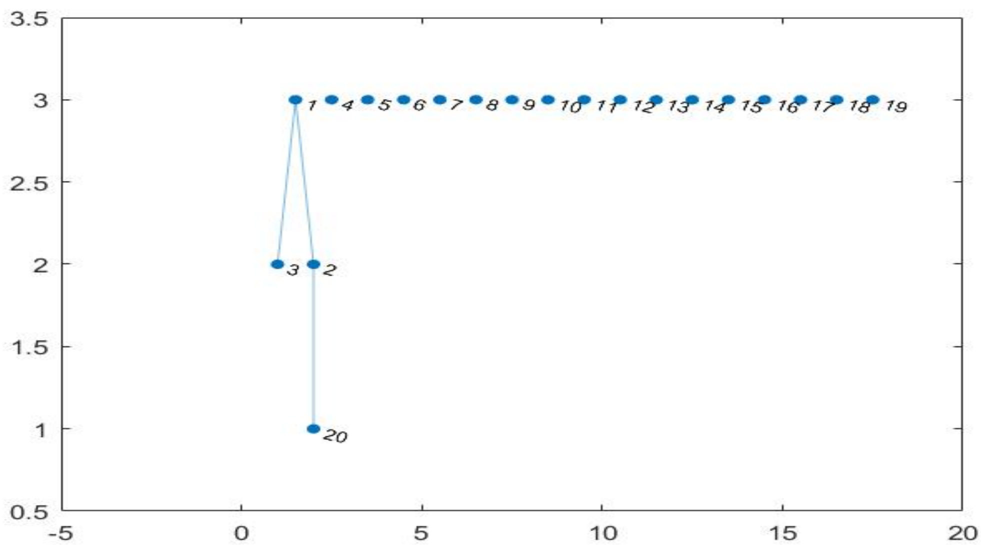

Figure 2.15 Designed unconnected topology from a star (optimal)

Subsequently, as in the case of the ring topology, we add an extra constraint for the topology to enhance a connected design and we derive the final topology depicted in Figure 2.16. It has 47 edges and it differs from the initial one on 42 edges.

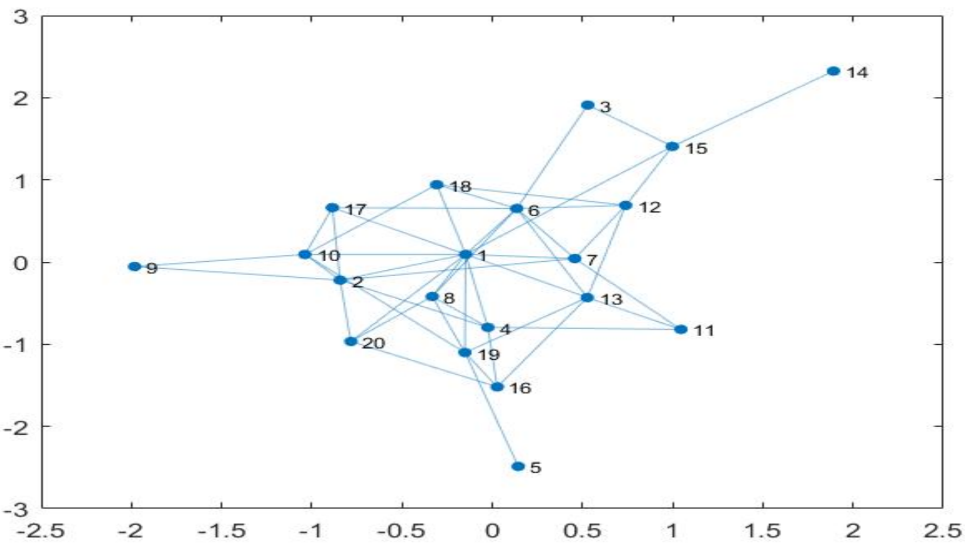

Figure 2.16 Designed connected topology from a star

Comments: From the study of these special structures we deduce several interesting conclusions. At first, in the case of topologies with very few edges, such as a ring or a star graph, the isolation of some agents from the rest network is sometimes optimal as it effectively stops their manipulative activity. The fact is that such a design may not be acceptable by these agents and by the society. So, we add more constraints, which do not affect the difficulty of the problem, in order to avoid a design which may be optimal but inapplicable in social networks. Fortunately, since the increase of the agent's friends leads to the decrease of her ability to manipulate each one of them, as we deduce from the sufficient condition (2.17), it is guaranteed that there exists another topology with more edges than the initial, which satisfies the stability constraints and it is in fact a local minimum of our optimisation problem. We can also design this topology to be connected by adding more edges and not affecting its stability.

2.5 Conclusion and Extensions

The main contributions of this work are a) the enrichment of the model for social choice procedures proposed in [38] by considering dynamically changing opinions and thus resulting in second order dynamics, b) the new approach for the stabilization of these dynamics through the graph topology design, which results in an integer programming problem with a BMI constraint and c) a proper genetic algorithm for the solution of this problem.

There are some interesting directions for future research, that is a) the study of the topology design problem for the case of weighted graphs and the case of networks with switching topology, b) the study of the effect of the graph topology on the equilibrium of the system and c) the examination of other techniques to solve the nonconvex integer optimisation problem and the implementation of comparative studies with our approach.

Chapter 3

Games of Social Distancing during an Epidemic

3.1 Introduction

The emergence of the Covid-19 pandemic is one of the most significant events of this era. It affects many sectors of human daily life and psychology. It indicates the inefficiency of many health care systems and it leads to state interventions in the functioning of the society through urgent measures and to economic depression. Especially at the beginning of the pandemic, non pharmaceutical methods were used on a large scale to contain its outspread. This happens because the behavioral changes of the agents can have significant effects on the delay and the prevalence of the epidemic. So, the central authorities, governments and health organisations, give guidelines and rules in order to induce social distancing and apply regional quarantines in many cases. However, it is up to the individuals to follow these rules, so the choice for social distancing can be modeled as a Nash game.

A lot of research has been conducted recently in the field of social distancing during an epidemic [15, 115, 96, 7, 30, 55, 5, 59, 34, 78, 114, 21, 26, 17, 9, 47, 16, 27, 28, 3, 60, 71].The main compartments of these works are a model for the spread epidemic and a game model for the decision making. Two well-organised surveys of game theoretic models for these issues are [62] and [22]. From the analysis of such models we obtain insights for the evolution of the epidemic and the human response to it and derive conclusions for the policies that should be followed and their consequences.

For the epidemic modelling almost all of the aforementioned references use compartmental models e.g., the SIR model. These models were introduced a century ago [101],[66] and they are well studied. Some works use more recent variations of the classic compartmental models, such as [78] where a spatial compartmental model is studied and [5, 30, 17] where the population is considered to consist of many types or classes with respect to the agents age, number of contacts etc, with each of these classes having a compartmental model with different parameters. Alternative approaches for the epidemic modelling are the percolation theory on networks [93, 90, 87, 46, 105] and the agent-based models [21, 35, 29, 33, 52], which emphasize more on the structure of human interconnections that affects the transmission of the disease.

The application of quarantines and social distancing has effects both on the economy and on human psychology, so the decisions for the measures to be followed concerns both a central policy maker and all the agents of the society. The works that focus on the decisions of a central policy maker model the social distancing as a control problem [78, 21, 17, 47, 16, 27].On the other hand, the works that focus on the agents' response to the epidemic outspread are considering game theoretic models [15, 115, 96, 7, 30, 55, 5, 59, 34, 114, 26], the majority of which are mean-field dynamic games. Finally, [9] is an interesting Stackelberg game approach combining both a central policy maker and many social agents.

Following these lines of research, in this work we consider an agent-based model for the epidemic outspread and a Nash game for the agents' response to the epidemic. We consider that each agent has a personal health state evolving in discrete time. The possible infection arises from her interactions with her neighbors. This agent-based model, similar to [59, 21], is a discrete analogue for the SIR model on networks, where local interactions play a major role. For the decision making, we consider that the agents choose their interactions as a trade-off between the danger of infection and the utility they earn from their social contacts. The agents are considered myopic, so the model studied is a sequence of static games.

A main characteristic of our model is the information that the agents possess during the decision making. We study two cases, the case of local information, where we assume that each agent has perfect knowledge for the health states of herself and her neighbors and the case of statistical information, where we assume that she knows only some indexes that describe the overall prevalence of the epidemic. The available information during a decision making procedure is a core issue in game theory and the role of information in the decision of an agent to apply social distancing has been examined also in [53] and [122], where the authors consider the spreading of word of mouth in social networks and its effect on the agents' behavior and the epidemics spreading.

The main contribution of this work is to introduce a model for the decision making procedure of the agents to apply spontaneous social distancing during the outbreak of an epidemic, which takes under account the networked structure of human interconnections. In this direction, the game formulated here concerns the local interactions of neighboring agents and it is not a mean-field game between the agents and the social average, contrary to most of the aforementioned references on social distancing modelling. Few recent works [5, 30] take into account the networked structure of human interconnections, and specifically its degree distribution, attempting however a mean-field asymptotic approach to work with the well established compartmental models. We should also point that our game model for the choice of social interactions is conceptually similar to the activation game model of [59], but in our formulation the agents choose each active interaction in a strategic manner. Moreover, our game takes place on a fixed network of social contacts -this way we can study various topologies- while the authors of [59] consider new random networks at each time step. Through the analysis of the introduced game theoretic model, we indicate the significance of the available information on the decisions for social distancing, which is a novelty of this work. Through that analysis, we compute the Nash equilibrium strategies and investigate their characteristics through numerical simulations. Finally, we study some variations of our initial problem, such as experimentation on various network types, the impact of fake information and of the finite capacity of a health care system and related simulations are presented and annotated.

3.2 The model

3.2.1 Notation

We denote by $G = (V,E)$ an undirected graph, where $V = \{1,...,n\}$ is the set of its nodes representing the agents and $E \subset V \times V$ is the set of its edges indicating the social relations between the agents. The sparsity pattern of this graph indicates the established social relations of each agent -family, friends, colleagues etc- with whom we assume she interacts. The social relations graph G changes very slowly compared to the epidemic dynamics, so we assume it to be constant during the time horizon of the epidemic. $A = \{a_{ij}\}$ is the adjacency matrix of the graph i.e., $a_{ij} = 1$ if $(i, j) \in E$, otherwise $a_{ij} = 0$. $N_i = \{j : (i, j) \in E\}$ is the neighborhood of agent i, and $\bar{N}_i = N_i \cup \{i\}$. $d_i = \sum_{j\in N_i} a_{ij}$ is the degree of node i, that is the number of her neighbors. We consider also a matrix $S = \{s_{ij}\}$, with the same sparsity pattern with the adjacency matrix A, which indicates the desire of each agent to meet with each one of her neighbors. The vector $\mathbb{0}_n$ is a vector of n zeros and the vector $\mathbb{1}_n$ is a vector of n ones. The logical or is noted by $\vee$ and the logical and by $\wedge$.

3.2.2 Actions

We consider a social distancing game, which is repeated at each day during the outspread of an epidemic. The actions of an agent i determine the intensity of the relations she wants to have with each one of her neighbors. So, the action of agent i at day k is a vector of length equal to the number of her neighbors given by:

$$u^i(k) = [u^i_{j_1}(k) \ldots u^i_{j_{d_i}}(k)] \in [0,1]^{d_i}, \tag{3.1}$$

where:

$$\{j_1, \ldots, j_{d_i}\} = N_i. \tag{3.2}$$

Each $u^i_j(k)$ indicates the desire of agent i to meet her neighbor j at day k. We assume that the intensity of the contact between the agents is proportional to their mutual desire to meet each other. For example, family members or sexual partners often have a great desire to meet each other and have a close contact, while friends or colleagues may not have the same desire to meet each other and even if they meet they can easily keep safe distances. According to the actions chosen by the agents we have an induced weighted adjacency matrix $W(k) = [w_{ij}(k)]$ for the network, which indicates the intensity of the contact between two neighbors at day k, where $w_{ij}(k)$ have the following form:

$$w_{ij}(k) = \begin{cases} 0 & \text{,if} \quad a_{ij} = 0 \\ u^i_j(k) u^j_i(k) & \text{,if} \quad a_{ij} = 1 \end{cases} \tag{3.3}$$

3.2.3 States

We consider that each agent has a health state consisted of two variables $x_i(k)$, which indicates if the agent has been infected before day k and $r_i(k)$, which indicates the duration of her infection and consequently if she has recovered. Here we assume that all the infected agents recover after R days. This assumption is made for simplicity of the model. The following analysis holds also for varying recovery period.

The vector $x^0 = [x^0_i]$ indicates the initial conditions for the x_i state of the agents. The probability p^0_x indicates the distribution of the initial conditions, which are assumed to be i.i.d. random variables:

$$x^0_i = \begin{cases} 0 & \text{,w.p.} \quad 1 - p^0_x \\ 1 & \text{,w.p.} \quad p^0_x \end{cases} \tag{3.4}$$

Remark 6. *The assumption that the initial health states of the agents can be modeled as i.i.d. random variables does not exactly hold for the study of any phase of the outbreak of* *an epidemic, since there exist correlations among the health states of the agents, imposed by the networked structure of their contacts. However, if we study the beginning of the outbreak in a community, where the initial number of infected agents is very small and they could have been infected through contacts with persons out of that community, there is no necessarily correlation among their health states and they can be described as independent random variables. The reason for the fact that we assume an identical distribution for the initial states of our population is that, with no extra information about each agent's past behavior, we cannot distinguish any individual of the population and assume a personalised distribution for her state.*

The vector $r^0 = \mathbb{0}_n$ indicates the initial conditions for the r_i state of the agents.

These states evolve as follows:

$$x_i(k+1) = \begin{cases} x_i(k) & ,\text{w.p.} \quad p_{x_i} \\ 1 & ,otherwise \end{cases} \tag{3.5}$$

where $p_{x_i} = \prod_{j \in N_i}(1 - w_{ij}(k)p^c x_j(k)\mathcal{X}_{\{r_j(k)<R\}})$ and p^c is the infection probability.

$$r_i(k+1) = \begin{cases} r_i(k) + x_i(k) & ,\text{if} \quad r_i(k) < R \\ R & ,\text{if} \quad r_i(k) = R \end{cases} \tag{3.6}$$

where R is the duration of the recovery period.

The probabilities $w_{ij}(k)p^c x_j(k)\mathcal{X}_{\{r_j(k)<R\}}$ indicate the possibility to have a meeting at day k and get infected by another agent. That agent can transmit the disease if she has been infected $(x_j(k) = 1)$ and has not recovered yet $(r_j(k) < R)$, which is shown with the use of the characteristic function:

$$\mathcal{X}_{\{r_j(k)<R\}} = \begin{cases} 1 & ,\text{if} \quad r_j(k) < R \\ 0 & ,\text{if} \quad r_j(k) = R \end{cases} \tag{3.7}$$

Remark 7. *In this simple model, which is a discrete analogue of the SIR model on graphs, we assume that every infected agent recovers. That is to avoid changes in the graph topology, which would make the analysis of the game more difficult. We expect this to cause minor differences in the case of an epidemic with low mortality.*

In order to model the probable infection of an agent j by her neighbor agent i, we use a similar formulation with the mean field approach [10], where the infection probability can be expressed as a function of the well known basic reproduction number R_0.

$$p^c(R_0) = 1 - (1 - \frac{R_0}{\bar{d}})^{\frac{1}{R}}. \tag{3.8}$$

Using this infection probability, for a large Erdős-Rényi random graph - that is the network analogue of a well mixed population - the continuous mean field approach of these agent dynamics, in the case of no social distancing, is the well known SIR model (e.g., [2]). Similar derivations for the probabilities that govern the transmission of the disease over networks of interconnected agents are existing in the relevant bibliography, such as [93].

3.2.4 Payoffs

We assume that the agents choose their actions, based on the available information, by maximizing their short-term payoffs. These payoffs are considered to depend solely on the benefits from the social interactions between the agents and on the costs to their health due to possible infection. In reality, the decision of a behavioral change depends also on socioeconomic and ethical considerations, which are omitted in this first approach, for the sake of simplicity.

So, in our case the instantaneous payoffs depend on two terms. The first one indicates the satisfaction that each agent derives by the interaction with her neighbors, these benefits differ between her neighbors. The second term shows the costs an agent suffers if she has been infected. Since the agent does not know her health state the next day, she tries to estimate it based on the available information the current day k, denoted by $I_i(k)$. The parameters G_i indicate the importance of the infection for each agent. We divide the agents into two groups: the vulnerable (large G_i) and the ones who are non-vulnerable (small G_i). The payoffs are given by:

$$J_i^k(I_i(k), u_j(k), j \in \bar{N}_i) = \sum_{j \in N_i} s_{ij} u_j^i(k) u_i^j(k) - G_i E\{x_i(k+1)|I_i(k)\} \mathcal{X}_{\{r_i(k+1)<R\}} \tag{3.9}$$

where $s_{ij} > 0$ is the payoff agent i derives from the interaction with agent j if $w_{ij}(k) = 1$. The actions are functions of the available information:

$$u^i(k) = \gamma_i(I_i(k)) \tag{3.10}$$

Remark 8. *The game situation is clearly dynamic.The actions of each player have long-term effects on both the epidemics and her future payoffs. However, each agent is difficult to predict the long-term effects of her actions and the evolution of the epidemics is highly uncertain, since many crucial factors of its dynamics are not known (e.g. seasonality [18], future pharmaceutics, mutations etc). So, we restrict ourselves to a model with myopic players.*

Moreover, we assume that the agents have bounded rationality, thus, they are not able to reason based on the history of their neighbors' actions nor to use such an information to infer conclusions for the correlations among their health states. In fact, these correlations are studied by the specialised doctors on the field of epidemiology to predict the evolution of the epidemic, but we believe that it is rather pretentious to assume that the majority of not specialised civilians make this kind of inferences to decide their daily actions.

3.3 Perfect local state feedback information

In this section, we study the case where the agents have perfect local state feedback information. That is, agents know exactly their current health state and the current health states of their neighbors before taking the decision to meet them or not. We denote this information structure I^F.

$$I_i^F(k) = \{x_j(k), r_j(k) : j \in \bar{N}_i\}. \tag{3.11}$$

In order to analyze the social distancing game under the perfect local state feedback information (3.11), we follow a step-wise analysis, considering a static, one-step game. All the time indices, indicating the days, will be omitted during this analysis. Instead, we will use the notation x_i^+ and r_i^+ for the next states. Based on the information (3.11), we can explicitly calculate the conditional expectation of each agent's next state $E\{x_i^+|I_i\}$:

$$E\{x_i^+|I_i\} = x_i \prod_{j\in N_i}(1 - w_{ij}p^c x_j \mathcal{X}_{\{r_j<R\}}) + (1 - \prod_{j\in N_i}(1 - w_{ij}p^c x_j \mathcal{X}_{\{r_j<R\}})) \tag{3.12}$$

since from (3.10) the strategies are measurable on the sigma fields defined by x, so $E\{u_j^i|x\} = u_j^i$. Thus, the payoffs have the following form:

$$J_i = \sum_{j\in N_i} s_{ij} u_j^i u_i^j - \left[G_i(x_i - 1) \prod_{j\in N_i}(1 - w_{ij}p^c x_j \mathcal{X}_{\{r_j<R\}}) + G_i \right] \mathcal{X}_{\{r_i^+<R\}}. \tag{3.13}$$

Proposition 2. *The strategy profile $u = \mathbb{0}_{\sum d_i}$ is a Nash equilibrium for the game with perfect local state feedback, since it results to indifference for all the agents.*

However, it is possible that there exist other Nash equilibria. At first, we prove in the following proposition that no strict equilibria can be found in the interior of the action space.

Proposition 3. *The best response of each agent always contains a point in* $\{0,1\}^{d_i}$*, i.e. the vertices of the action space. Therefore, there is no strict Nash equilibrium in* $[0,1]^{\Sigma d_i} \setminus \{0,1\}^{\Sigma d_i}$

Proof. We calculate the first and second partial derivatives of J_i:

$$\frac{\partial J_i}{\partial u^i_j} = u^j_i \left[s_{ij} + G_i(x_i - 1)\mathcal{X}_{\{r_i^+ < R\}} p^c x_j \mathcal{X}_{\{r_j < R\}} \prod_{k \in N_i \setminus \{j\}} (1 - u^i_k u^k_i p^c x_k \mathcal{X}_{\{r_k < R\}})\right] \tag{3.14}$$

$$\frac{\partial^2 J_i}{(\partial u^i_j)^2} = 0 \tag{3.15}$$

for all $j \in N_i$, so:

$$\nabla^2 J_i = 0 \tag{3.16}$$

and thus J_i is a harmonic function. So, form the maximum principle for harmonic functions on compact sets ([100] chapter 4) we conclude that the local maxima of J_i with respect to u_i are on the boundary of $[0,1]^{d_i}$. Applying successively the maximum principle for the faces and the edges of the hypercube $[0,1]^{d_i}$, observing that J_i is still harmonic on each face of the hypercube with respect to the free variables on that face (the u^i_j that are not fixed to 0 or 1), we conclude that the best response of each agent always contains a point in $\{0,1\}^{d_i}$. □

Remark 9. *If agent i is infected,* $x_i = 1$ *and* $r_i < R$*, then* $J_i = \sum_{j \in N_i} s_{ij} u^i_j u^j_i - G_i$ *and if she has been recovered,* $r_i = R$*, it is assumed that she cannot get infected again. So, in these cases, an optimal strategy for her is* $u^i_j = 1, \forall j \in N_i$*, since if* $u^j_i = 1 \implies u^i_j = 1$ *and if* $u^j_i = 0$ *she is indifferent so she can also choose* $u^i_j = 1$.

Remark 10. *If agent i and agent j are neighbors and agent i is not infected (*$x_i = 0$*) and agent j is not infected (*$x_j = 0$*) or recovered (*$r_j = R$*) the optimal strategies for their interaction are* $u^i_j = 1$ *and* $u^j_i = 1$*, since if* $u^j_i = 1$*:* $J_i(u^i_j = 1) - J_i(u^i_j = 0) = s_{ij} > 0$ *and if* $u^i_j = 1$*:* $J_j(u^j_i = 1) - J_j(u^j_i = 0) = s_{ji} > 0$.

So defining the following sets:

$$\text{Inf}_i = \{j \in N_i : x_j = 1, r_j < R\} \tag{3.17}$$

and $|\text{Inf}_i|$ is the number of elements of Inf_i, we conclude that:

$$J_i = J_i(u^i_j : j \in \text{Inf}_i), \tag{3.18}$$

since the rest strategies are fixed. In this case, the computation of the equilibrium strategies is a single objective, multi-variable, integer optimization problem for each agent, which can be solved easily using the following algorithm for each agent in $O(|\mathrm{Inf}_i|(\log(|\mathrm{Inf}_i|)+1))$ iterations:

Algorithm 1 Solution of the optimization problem for each agent

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The optimal strategies $(u^i_j)^*$ for $j \in \mathrm{Inf}_i$
Sort the parameters s_{ij}, $j \in \mathrm{Inf}_i$ in decreasing order
Define the sequence of indices $j_1 \dots j_{|\mathrm{Inf}_i|}$ to be the j-indices of the previous ordering
Define the strategies $\bar{u^i_0} = \mathbb{0}_{\mathrm{Inf}_i}$, $\bar{u^i}_k = \{u^i_{j_1} = 1 \dots u^i_{j_k} = 1, u^i_{j_{k+1}} = 0 \dots u^i_{j_{|\mathrm{Inf}_i|}=0}\}$, $k = 1 \dots |\mathrm{Inf}_i|$
$k = 0$
$\Delta J_i = 1$
while $\Delta J_i > 0$ and $k \leq |\mathrm{Inf}_i|$ **do**
    $\Delta J_i = s_{ij_k} - G_i p^c (1-p^c)^k$
    $k = k+1$
end while
$(u^i_j)^* = \bar{u^i}_{k-1}(j_k = j)$

Remark 11. *Each player may implement Algorithm 1 independently of the others. Thus, the players can reach the Nash equilibrium in a decentralized way.*

Remark 12. *The strategy profile* $u^i_j = \max\{x_i, 1-x_j\}$ *is a Nash equilibrium for the game with perfect local state feedback* (3.11)*, if* $\forall i \notin \bigcup Inf_i : \max\{s_{ij} : j \in Inf_i\} < G_i p^c$

This equilibrium shows the phenomenon that in the case the agents are highly vulnerable to the disease and they know the state of their neighbors, they communicate with all the healthy ones in order to maximize their payoffs and the infected try to communicate also with their neighbors for the same reason but they are banned by them. So, this equilibrium results to higher payoffs for the non infected agents:

$$J_i = \begin{cases} \sum_{j \in N_i} s_{ij}(1 - x_j \mathcal{X}_{\{r_j < R\}}) & , x_i = 0 \vee r_i = R \\ -G_i & , x_i = 1 \wedge r_i < R \end{cases} \tag{3.19}$$

3.4 Statistical Information

The second case that we study is the case where the agents have statistical information for the distribution of the states. Our motivation for studying this case is that in the first stage of the COVID-19 pandemic the diagnostic tests were not available for everyone and in the current stage of the pandemic many infected agents are asymptomatic and they do not proceed to continuous testing. So, the agents get informed from the media just for the prevalence of the disease and they ignore the health state of each one of their neighbors and their own health state as well. However, due to this lack of information imposed on the agents the analysis of the game become far more complex and we have to make several simplifying assumptions to deal with it.

At first, we assume that the agents ignore any correlations among their states, so they perceive that their states follow a Bernoulli distribution. As we have stated in subsection , this is a common assumption connecting the graph theoretic models with the SIR model. We assume also that all the agents know the same distribution with the same parameters - which holds if this information is broadcasted - and that they have no memory for the past values of these parameters:

$$I_i^S(k) = \{p_x(k), p_r(k)\}, \tag{3.20}$$

where

$$p_x(k) = \frac{|\{i : 1 \leq r_i(k) < R\}|}{N}, \tag{3.21}$$

is the percentage of infected agents at day k and

$$p_r(k) = \frac{|\{i : r_i(k) = R\}|}{N}, \tag{3.22}$$

is the percentage of recovered agents at day k.

Furthermore, we assume that each agent chooses the same probability to meet each one of her neighbors and then makes d_i random experiments to decide if she will meet each one of them.

$$u_j^i(k) = \begin{cases} 1 & \text{, w.p.} \quad p_u^i(k) \\ 0 & \text{, otherwise} \end{cases} \tag{3.23}$$

This is rational only if the utility earned from each interaction is the same from all the neighbors of each agent: $s_{ij} = s_i, \forall j \in N_i$. We assume that this symmetry holds for this case.

Remark 13. *We have to point out here, in order to avoid confusion, that this problem formulation is slightly different than the one presented in section 2. In both cases, the actions* $u_j^i(k)$ *indicate the intensity of a contact, which, motivated by the results of section 3, is either 0 (no meeting) or 1 (meeting). However, in this case the occurrence of a meeting is considered a random event with probability* $p_u^i(k)$*. The reason for this modeling is that the agent is not able to differentiate among her neighbors, because the danger to be infected as well as the pleasure earned form the interaction are assumed to be the same. Thus, she makes* d_i *independent random experiments which determine who she will meet.*

Consequently, the strategy space of each agent is:

$$p_u^i(k) \in [0,1]. \tag{3.24}$$

We then drop k in order to proceed with the analysis of one step of the game. In order to study the equilibria of this game we need the expectation of the state of the agents based on the available information (3.20). Thus, we compute at first the expectation of the next state of an agent given the current states:

$$E\{x_i^+ \mid x,r\} = 1-(1-x_i)\prod_{j\in N_i}(1-u_j^i u_i^j p^c x_j \mathcal{X}_{(r_j<R)}), \tag{3.25}$$

next we compute the expectation of the previous conditional expectation over all the states:

$$E_{x,r}\big\{E\{x_i^+ \mid x,r\}\big\} = 1-(1-p_x)\prod_{j\in N_i}(1-u_j^i u_i^j p^c p_x(1-p_r)), \tag{3.26}$$

and thus the criteria have the following form:

$$J_i = s_i\sum_{j\in N_i} u_j^i u_i^j + \left[G_i(1-p_x)\prod_{j\in N_i}(1-u_j^i u_i^j p^c p_x(1-p_r)) - G_i\right](1-p_r), \tag{3.27}$$

where the strategies are random and uniform for all the neighbors of an agent according to eq.(3.23), so we have to compute the expected criteria, given the probabilities p_u^i of the uniform strategies:

$$\hat{J}_i = E\{J_i | p_u^i, p_u^j, j\in N_i\} = s_i p_u^i \sum_{j\in N_i} p_u^j + \big[G_i(1-p_x^0)\cdot\prod_{j\in N_i}(1-p_u^i p_u^j p^c p_x^0) - G_i\big](1-p_r) \tag{3.28}$$

Each agent wants to maximize $\hat{J}_i$ w.r.t. p_u^i.

Proposition 4. *The possible equilibria of the game with statistical information are in* $\{0,1\}^N$ *i.e., for each i,* p_u^i *is either* 0 *or* 1.

Proof. We compute the first two derivatives of $\hat{J}_i$ w.r.t. p_u^i we get:

$$\frac{\partial \hat{J}_i}{\partial p_u^i} = s_i\sum_{j\in N_i} p_u^j - G_i(1-p_x)(1-p_r)\cdot\sum_{j\in N_i} p_u^j p^c p_x(1-p_r)\prod_{k\in N_i\setminus\{j\}}(1-p_u^i p_u^k p^c p_x(1-p_r)), \tag{3.29}$$

and

$$\frac{\partial^2 \hat{J}_i}{(\partial p_u^i)^2} = G_i(1-p_x)(1-p_r)\sum_{j\in N_i} p_u^j p^c p_x(1-p_r) \cdot \sum_{k\in N_i\setminus\{j\}} p_u^k p^c p_x(1-p_r) \prod_{l\in N_i\setminus\{j,k\}} (1-p_u^i p_u^l p^c p_x(1-p_r)), \tag{3.30}$$

Note that $\frac{\partial^2 \hat{J}_i}{(\partial p_u^i)^2} \geq 0$. If $\hat{J}_i$ is strictly convex with respect to p_u^i, then its maximizer lies in $\{0,1\}$.
If $\frac{\partial^2 \hat{J}_i}{(\partial p_u^i)^2} = 0$, then all p_u^j in N_i have to be 0, except at most one of them. Indeed, all the other quantities in (3.30) are strictly positive before the end of the epidemic (when the epidemic ends $p_x = 0$). Particularly, $G_i(1-p_x)(1-p_r) > 0$, $p_x p^c > 0$, and $1 - p_u^i p_u^l p^c p_x > 0$. Let $p_u^{j*} \neq 0$. Then, a value $\tilde{p}_u^i$, with $0 < \tilde{p}_u^i < 1$, can be a best response for player i, only if:

$$\hat{J}_i(p_u^i = 1) = \hat{J}_i(p_u^i = 0) = 0. \tag{3.31}$$

Thus:

$$p_u^{j*}[s_i - G_i(1-p_r)(1-p_x)p^c p_x] = G_i(1-p_r)p_x, \tag{3.32}$$

and:

$$\frac{\partial \hat{J}_i}{\partial p_u^i}(p_u^i = 0) = 0, \tag{3.33}$$

which implies:

$$p_u^{j*}[s_i - G_i(1-p_r)(1-p_x)p^c p_x] = 0. \tag{3.34}$$

It is then obvious that, while the epidemic continues ($p_x > 0$), the equations (3.32) and (3.34) are contradictory. So, p_u^i has to be in $\{0,1\}$ for all i. □

In order to characterize the Nash equilibria of this game we observe that it is strategically equivalent to the following one:

$$\tilde{J}_i(p_u^i, p_u^{-i}) = a_i p_u^i \sum_{j\in N_i} p_u^j + \prod_{j\in N_i} (1 - b p_u^i p_u^j), \tag{3.35}$$

where:

$$a_i = \frac{s_i}{G_i(1-p_x)(1-p_r)}, \quad b = p^c p_x(1-p_r), \tag{3.36}$$

and $p_u^i \in \{0,1\}$, for all i.

We proceed with the calculation of the best response for each agent. From Proposition 4 we know that each agent plays $p_u^i = 0$ or $p_u^i = 1$, so we rewrite the payoffs of the agents as functions of the number of their neighbors playing $p_u^j = 1$. We denote this number m_i.

$$\tilde{J}_i(p_u^i, m_i) = a_i m_i p_u^i + (1 - b p_u^i)^{m_i} \tag{3.37}$$

and

$$\tilde{J}_i(0, m_i) = 1, \tag{3.38}$$

$$\tilde{J}_i(1, m_i) = a_i m_i + (1-b)^{m_i}. \tag{3.39}$$

Thus, her best response depends solely on m_i. To study this dependence, we define the following functions:

$$f_i(m) = \tilde{J}_i(1, m) = a_i m + (1-b)^m = a_i m + e^{m \ln(1-b)} \tag{3.40}$$

The best response of each agent is:

$$BR_i(m_i) = \begin{cases} 1 & \text{, if} \quad f_i(m_i) > 1 \\ 0 & \text{, otherwise} \end{cases} \tag{3.41}$$

So, we propose Algorithm 2 for the computation of the actions corresponding to a Nash equilibrium.

Algorithm 2 Computation of the NE strategies for the game with information for the distribution of the states

The optimal strategies p_u^{i*}
Set $p_u^i = 1, \forall i$
Compute $f_i(m_i), \forall i$ $(m_i = d_i)$
while $\exists f_i(m_i) \leq 1$ **do**
  if $f_i(m_i) \leq 1$ **then**
    Set $p_u^i = 0$
  end if
  Compute new $m_i, \forall i$
  Compute new $f_i(m_i), \forall i$
end while

Proposition 5. *There exists a Nash equilibrium of the game with statistical information for the distribution of the states. Furthermore, Algorithm 2 converges to the Nash equilibrium in $\mathcal{O}(N^2)$ steps.*

To prove this proposition we firstly prove the following lemma:

Lemma 2. *For the functions* $f_i(m)$*, defined in* (3.40)*, there exists a unique* $m_0 \in \mathbb{R}_+$ *such that* $f(m_0) = 1$ *and for all* $m > m_0$*,* $m \in \mathbb{N}$ *:* $f(m) > 1$.

Proof. It is easily observed that $f_i(m)$ is convex and $f_i(0) = 1$ for each i. So, if $f_i'(0) \geq 0 \Rightarrow f_i(m) > 1$, $\forall m$, in this case $m_0 = 0$. Else if $f_i'(0) \leq 0 \Rightarrow \exists! m_0 \in \mathbb{R}_+^* : f(m_0) = 1$ and $\forall m > m_0$, $m \in \mathbb{N} : f(m) > 1$ due to the convexity of $f_i(m)$. $\square$

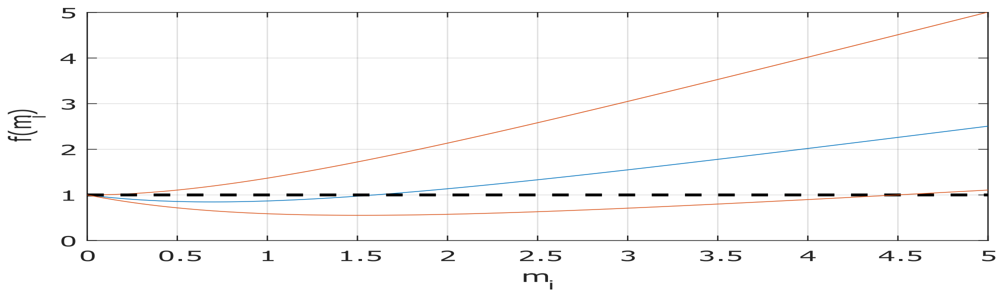

Figure 3.1 The function $f(m_i)$, for several values of a_i and $ln(1-b) = -1$.

Proof of the Proposition 5. : Due to this lemma, beginning with the maximum feasible value for m_i (which is d_i) the changes in the agents actions from 1 to 0 can result only in the decrease of their neighbors m_j's and thus it is possible to happen only one change of action $(1 \rightarrow 0)$ for each agent until the algorithm converges. To see this observe that if $f(m_i) \leq 1$ as m_i becomes smaller $f(m_i)$ cannot become larger than 1. Moreover, due to this observation, in the worst case the 'while-loop' will run N times and so the algorithm will converge in $\mathcal{O}(N^2)$ steps.

The point that the algorithm converges is a Nash equilibrium of the game, since the agents actions are their best responses to their active contacts numbers m_i's and for this profile of m_i's no agent will be benefited from a unilateral deviation from her action.

Furthermore, we should point that, since the algorithm is in fact a descent on the possible m_i-profiles, i.e. it initializes with all the contacts being active ($m_i = d_i$, $\forall i$) and each m_i decreases or stays the same, the Nash equilibrium that the algorithm converges is the one corresponding to the maximum possible sociability for the agents. $\square$

Remark 14. *Each player i, to implement Algorithm 2, needs to know the number of neighbors intended to play* $p_u^j = 1$ *i.e.,* m_i*. So after each iteration of the algorithm we assume that each player broadcasts to her neighbors her intended action, and finally chooses the actual* p_u^i *to play after Algorithm 2 converges.*

Remark 15. *If for each agent i it holds that* $s_i d_i + G_i(1-p_x)(1-p_r)[(1-p^c p_x(1-p_r))^{d_i} - 1] > 0$ *then the strategy profile* $p_u^i = 1, \forall i$ *is a Nash equilibrium of that game.*

Proposition 6. *The strategy profile* $p_u^i = 0, \forall i$ *is again a Nash equilibrium, since it results to indifference between the unilateral changes of each agent.*

The analysis of this section, and especially Proposition 4, indicates a rather interesting fact: in the statistical information game the agents choice is either full isolation or no social distancing at all. This phenomenon can be attributed to the fear of the agents due to the lack of knowledge about their neighbors' health states. If the prevalence of the disease is high and the agent considers herself to be vulnerable it is probable to be afraid to have any social interaction and choose full isolation. On the other hand, if the agent considers herself non-vulnerable or the prevalence of the disease to be low it is rather probable to continue her daily activities without applying social distancing.

3.5 Numerical studies

In this section we present several simulations for the social distancing games under the two different information structures in order to compare the disease prevalence and the agents payoffs in both cases. The payoffs of the agents have the form (3.9) at each day k, indicating the myopic behavior of the agents, who cannot predict the future consequences of their actions. The strategies considered are the Nash Equilibrium actions of the static games repeated each day of the epidemic outbreak.

For the game with perfect local information we consider that agent i plays $u_j^i(k)^* = 1$ if she has recovered or if her neighbor j is not infectious at day k and $u_j^i(k)^*$ to be the solution of Algorithm 1 otherwise. In the execution of Algorithm 1, we use the set $\text{Inf}_i = \{j \in N_i : 1 \leq r_j(k) < R\}$.

For the game with statistical information we consider the strategy profile $u^*(k)$, $k = 1, \dots, K$ to be the solution of Algorithm 2, where $p_x^0 = p_x(k)$ follows the rule (3.22).

The underlying graph topology is a random graph [36] with $N = 100$ agents, adjacency probability $p_{adj} = 10\%$ and average degree $\bar{d} = 10$. The recovery period is assumed to be 14 days. The sociability parameters s_{ij} are random numbers in $(0,1)$. The agents are divided into two groups the vulnerable and the non-vulnerable. For the vulnerable $G_i = 10000$ and for the non-vulnerable $G_i = 1000$. The percentage of the vulnerable in the community is 20%. The initial percentage of infected agents is 4%. The basic reproduction number of the disease is assumed to be $R_0 = 2.7$. Since all of the aforementioned parameters of our artificial agents are assigned at random, we use Monte Carlo method to obtain representative numerical results. So, all the simulations presented in this section and in the following section are the average of 100 Monte Carlo iterations.

In Figure 3.2 we show the effects of the social distancing games with perfect local information and with statistical information on the disease prevalence and on the sociability of the agents.

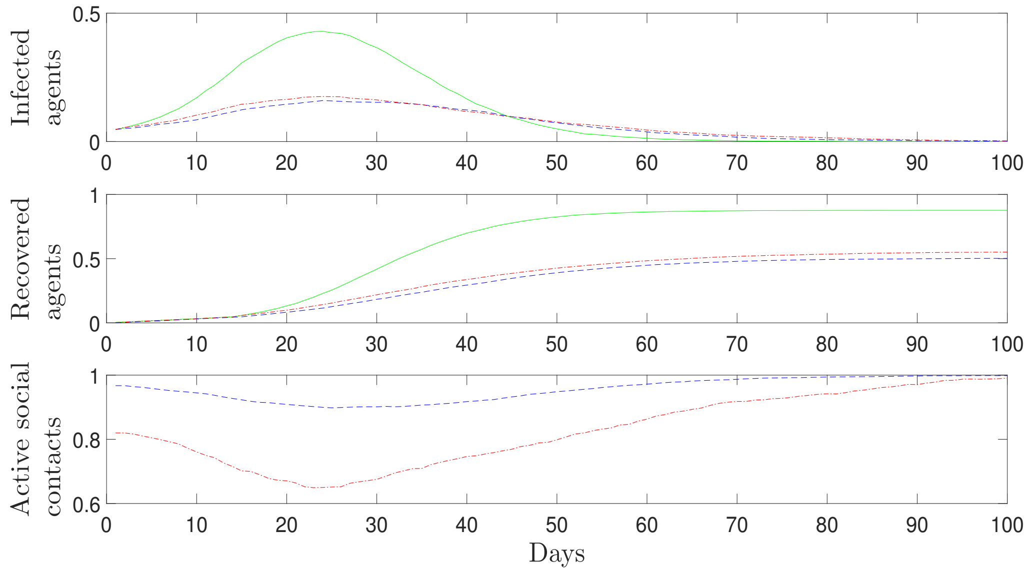

Figure 3.2 Infection, recovery and sociability curves for the case of no social distancing (solid line), the case of the game with perfect local information (dashed line) and the case of the game with statistical information (dashed-dotted line)

We observe that, for these values of the agents' parameters, both games result in similar results with respect to the diminishment of the epidemic outbreak. However, the different information affects significantly their strategies, since in the game with statistical information the agents are more cautious and apply strict social distancing due to the lack of knowledge of the health states of their neighbors.

We indicate the effects of the parameters of the agents criteria on the outspread of the epidemic in the following Figures 3.3 & 3.4. In these simulations we have considered that the parameters s_{ij} are bounded from 1 while the scale of the parameters G_i for the non vulnerable agents vary from 10 to 2000 and for the vulnerable agents is 10 times bigger. Thus, we consider several different ratios $r = \frac{\max\{s_{ij}\}}{G_i}$ and we observe their effects on the agents strategies and on the epidemic dynamics for both games.

From Figures 3.3 & 3.4, we clearly observe that the ratio of the sociality and vulnerability parameters plays a crucial role on the epidemic outspread as it models the effect of the trade off between fear of infection and socialisation on the agents behavior.

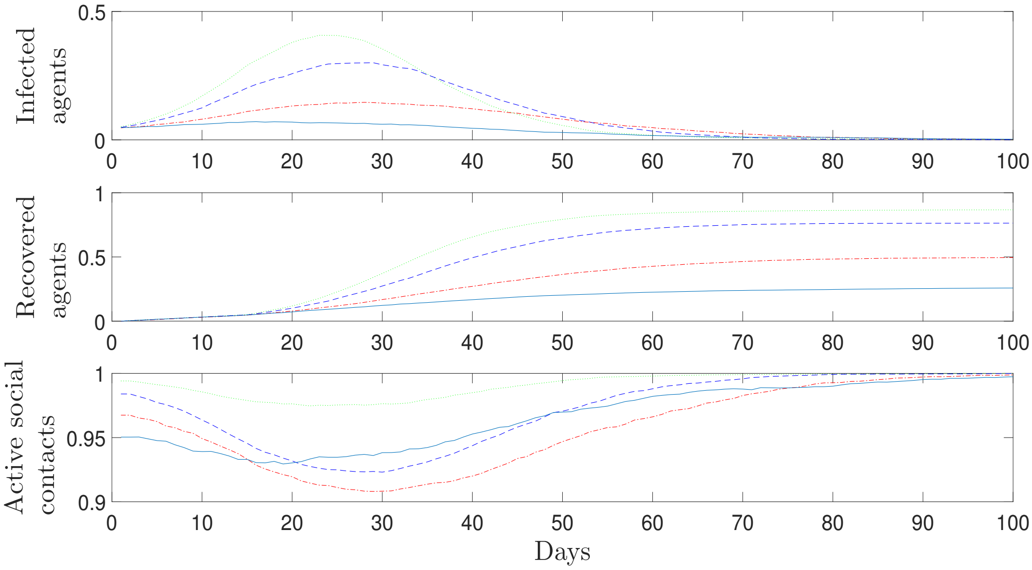

Figure 3.3 Infection, recovery and sociability curves for the game with perfect local information: $r = \frac{1}{100}$ (dotted line), $r = \frac{1}{300}$ (dashed line), $r = \frac{1}{1000}$ (dashed-dotted line) and $r = \frac{1}{2000}$ (solid line)

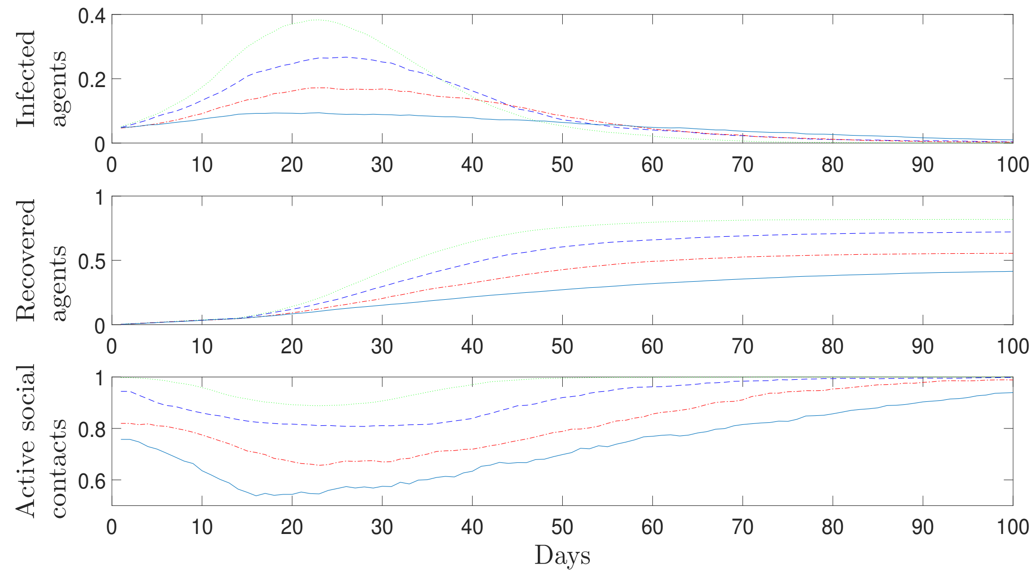

Figure 3.4 Infection, recovery and sociability curves for the game with statistical information: $r = \frac{1}{100}$ (dotted line), $r = \frac{1}{300}$ (dashed line), $r = \frac{1}{1000}$ (dashed-dotted line) and $r = \frac{1}{2000}$ (solid line)

Despite the fact that the limitation of the epidemic outspread is comparable in both games and depends strongly on the parameters of the agents' criteria, there is a remarkable difference on their actions, that is the social distancing they need to apply so as to achieve these goals. This difference on the agents behavior affects their payoffs. As we observe in Figure 3.5, the average payoff of the game with perfect local information is higher than the average payoff of the game with statistical information. Moreover, in the case of statistical information, both categories of agents suffer a loss in their payoffs due to the augmented social distancing, but the vulnerable agents suffer also because they are unable to choose rationally their social interactions and it is more probable for them to get infected. So, the vulnerable agents pay a greater burden for not being well informed.

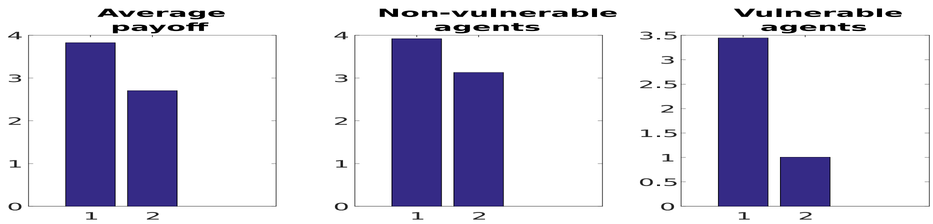

Figure 3.5 Comparison of the average payoffs of the agents for the two games. The case 1 stands for the game with perfect local information and the case 2 for the game with statistical information

3.6 Case Studies and Discussion

In this section, we consider several variations of the initial problem and examine, through simulations, the effects of the varying parameters on the behavior of the agents and on the outspread of the epidemic. All the results are based on Monte Carlo iterations and all the parameters, except the ones being under examination, are the same with the parameters of the previous section.

3.6.1 Effects of the graph topology on the outspread of the disease

At first, we study the effects of the topology of the underlying network, which represents the social interactions of the agents, on their behaviour and on the epidemic outspread. In Figure 3.6 we study the effects of the average degree i.e., the average number of neighbors of each agent, on the epidemic peak, on the total epidemic size and on the maximum social distancing i.e., the minimum active social contacts. We considered random graphs of 100 agents with varying adjacency probabilities $p_{adj} = 0.03, ..., 0.15$ resulting in average degrees $\bar{d} = 3, .., 15$. The increase of the average degree, which results in a better mixing of the population results in the increase of the total infection outspread for both games. Moreover,

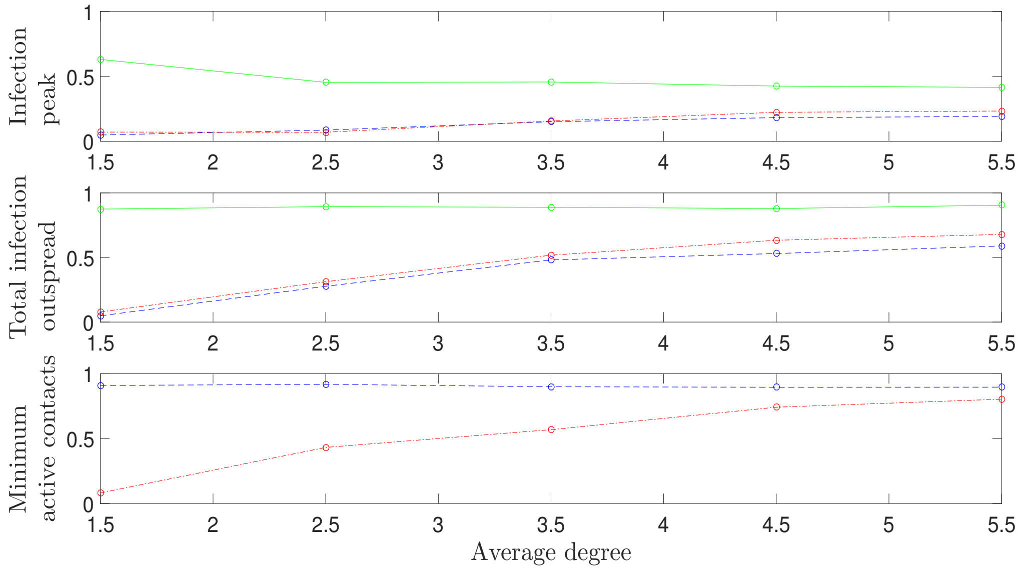

Figure 3.6 The effects of the average degree of the graph topology for the case of no social distancing (solid line), the case of the game with perfect local information (dashed line) and the case of the game with statistical information (dashed-dotted line)

in the case of the game with statistical information a small average degree results in almost isolation of the agents during the social distancing.

Subsequently, we present the effects of the graph topology. We consider four different graph topologies: a random graph [36], a stochastic block graph i.e., a coalition of 5 blocks (random graphs) with higher adjacency probability for the agents belonging to the same block, a scale free graph [11] and a small world graph [123]. In every case we have chosen the network parameters in a way that the graphs have almost the same average degree ($\bar{d} \approx 10$), in order to avoid the consequences of different degrees shown in Figure 3.6. In Figure 3.7, we present the effects of the topology in the case of no social distancing game. In Figure 3.8, we examine the case of the game with perfect local information and in Figure 3.9, the case of the game with statistical information.

In every case we observe that the topology affects both the epidemic outbreak and the agents behavior. The segmentation of the population into ill interconnected blocks (stochastic block graph) results in the diminishment of the outbreak in every case. The scale free property i.e., the existence of central nodes with significantly higher degree, results in an early high peak of the epidemic and the consequent need for strict social distancing during this period. Finally, the small world property i.e., the existence of edges that reduce the graph diameter, results in lower peaks but in extended duration of the epidemic and thus it results to the need for an extended "soft" social distancing.

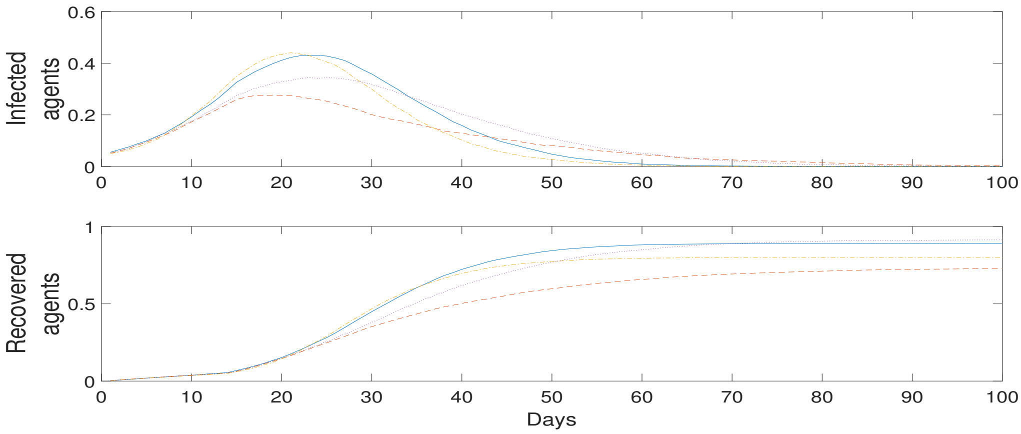

Figure 3.7 Epidemic dynamics for a random graph (solid line), a stochastic block graph (dashed line), a scale free graph (dashed-dotted line) and a small-world graph (dotted line) for the case of no social distancing

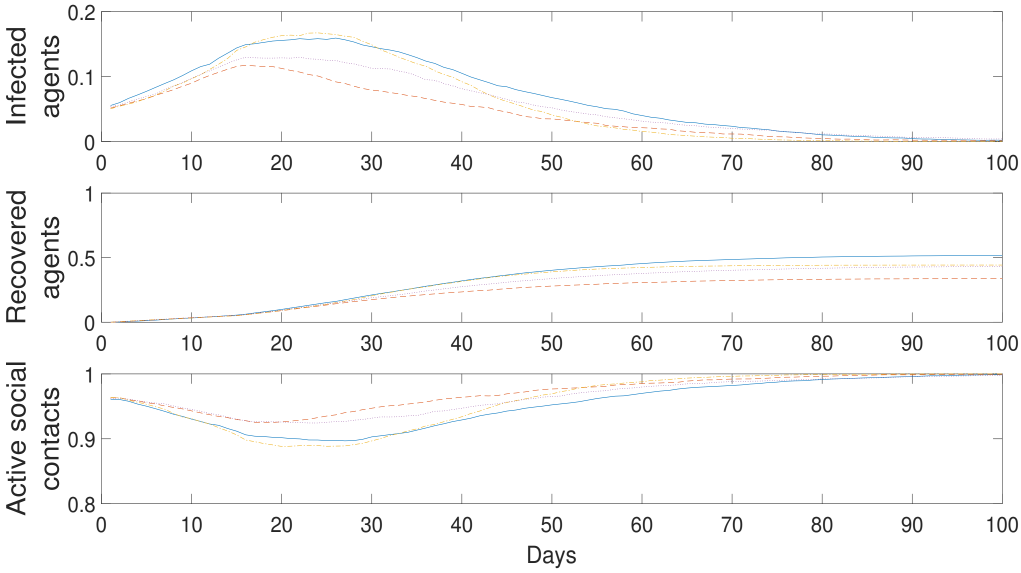

Figure 3.8 Epidemic dynamics for a random graph (solid line), a stochastic block graph (dashed line), a scale free graph (dashed-dotted line) and a small-world graph (dotted line) for the game with perfect local information

3.6.2 Virus transmissibility

A very important characteristic of every epidemic disease is its transmissibility. In the compartmental models the transmissibility is incorporated in the basic reproduction number R_0. So, in this subsection we study the effects of the parameter R_0 on the epidemic outspread and on the agents behavior for both games, with perfect local and with statistical information.

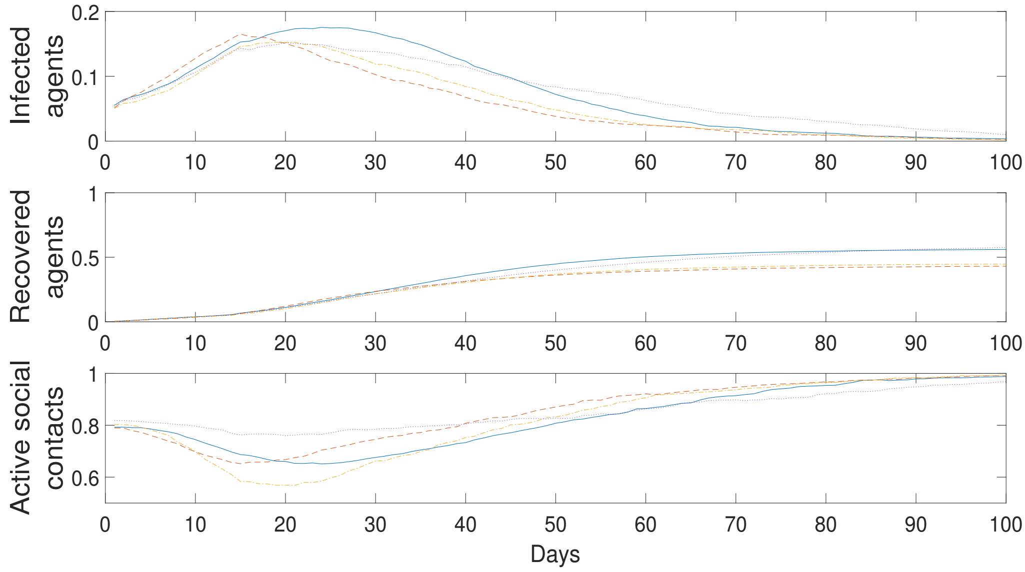

Figure 3.9 Epidemic dynamics for a random graph (solid line), a stochastic block graph (dashed line), a scale free graph (dashed-dotted line) and a small-world graph (dotted line) for the game with statistical information

We consider $R_0 = 1.5, \ldots, 5.5$. From Figure 3.10 we observe that in the case of no social

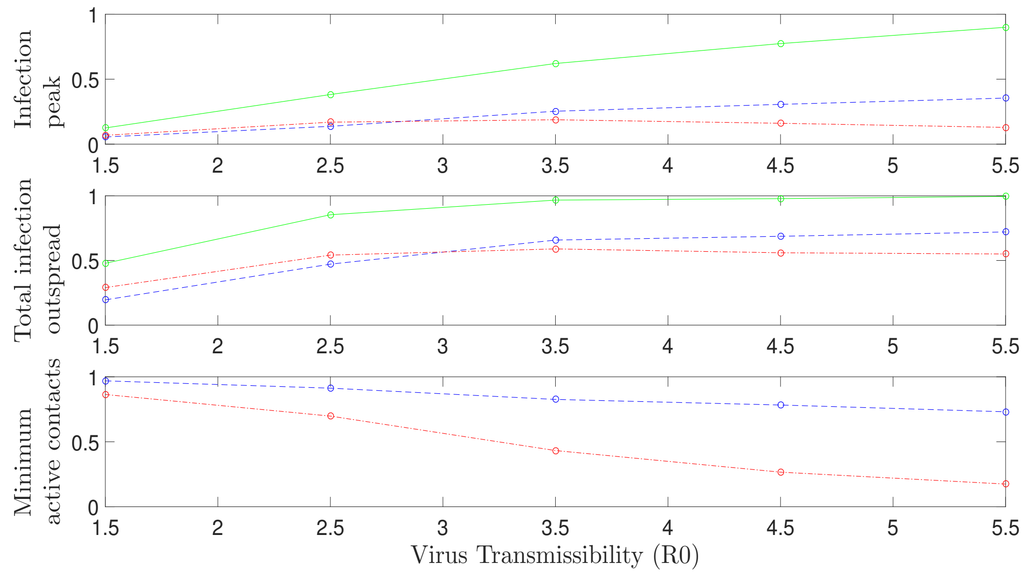

Figure 3.10 The effects of the virus transmissibility for the case of no social distancing (solid line), the case of the game with perfect local information (dashed line) and the case of the game with statistical information (dashed-dotted line).

distancing and in the case of the game with perfect local information the disease prevalence (peak and total size) is increasing with respect to R_0. However, in the case of the game with statistical information the disease prevalence remains the same for $R_0 \geq 2.5$, but with a high effort on social distancing which increases as R_0 increases. This indicates that in the case of statistical information the agents seem to fear a highly transmissible disease and apply strict social distancing.

3.6.3 The role of vulnerable agents

The vulnerable agents can be considered as key players for both games, since they tend to play conservatively and thus enhance the social distancing. In Figure 3.11, we show the effect of the percentage of vulnerable agents in the community to the infection peak and to the total number of infected agents for the game with perfect local information and in Figure 3.12, we show the same effects for the game with statistical information. The red lines are the linear regression curves for our experiments on different percentages and indicate the negative correlation of the percentage of vulnerable agents with the infection outspread for both games.

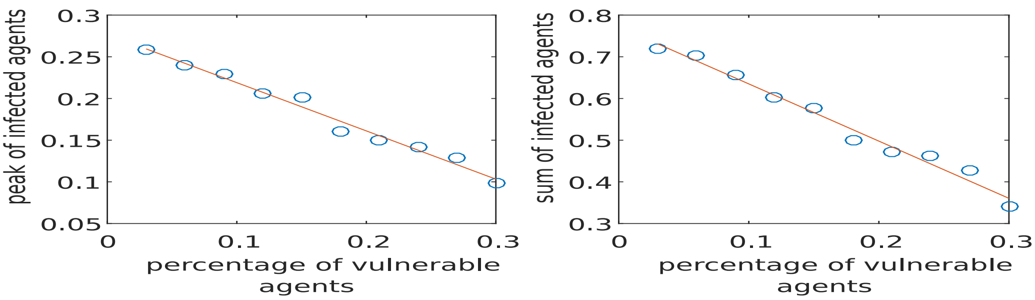

Figure 3.11 Correlation of the percentage of vulnerable agents with the infection outspread for the game with perfect local information.

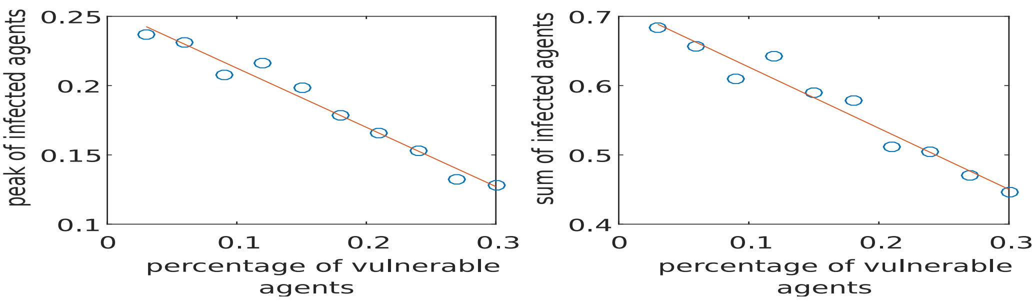

Figure 3.12 Correlation of the percentage of vulnerable agents with the infection outspread for the game with statistical information.

Next, we study a variation of the game with statistical information, where the vulnerability parameters of the agents (G_i) depend on the level of infection in the community. This is a realistic scenario, since the health systems worldwide have finite and usually small capacity, so if the number of infected agents who need health care pass a certain level it is not probable for the next agents who will get infected to have access in the necessary facilities. We model this phenomenon considering the vulnerability parameters to be proportional to the infection ratio:

$$G_i = G_i(p_x) = G_i^0 \alpha p_x \tag{3.42}$$

where G_i^0 are the constant vulnerability parameters used in all the previous simulations. Choosing $\alpha = \frac{1}{p_x^{\text{ref}}}$ we can define a reference infection level p_x^{ref}, where the agents will play as in the case of constant vulnerability parameters G_i^0. Below this level, they will be more indifferent for the effects of the disease on them and care more for their social interactions and above this level they will be more worried about the disease and follow social distancing strategies. This is confirmed by Figure: 3.13, where $p_x^{\text{ref}} = 8\%$

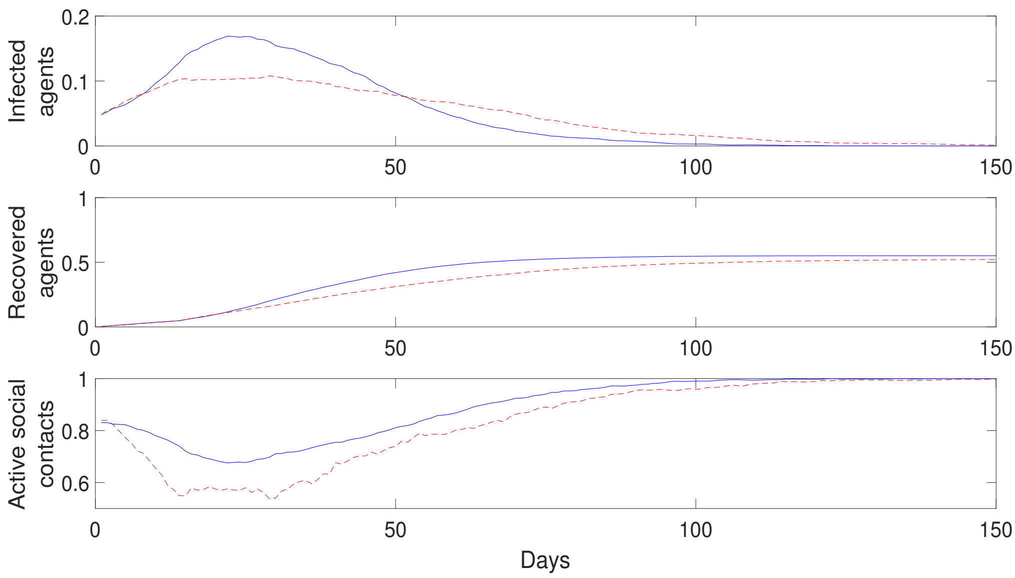

Figure 3.13 Infection, recovery and sociability curves when the vulnerability parameters are constant (solid line) and when they have a proportional dependence on the infection outspread (dashed line).

3.6.4 Fake statistical information

Finally, we study a modified scenario for the game with statistical information where we assume that the information the agents possess is fake or biased. This scenario is interesting because the agents usually get informed through mass media or social media. Consequently, the information they get is usually exaggerated or understated. The spread of fake news is another factor affecting the information quality. Moreover, in many cases the lack of diagnostic tests in the community makes the knowledge of the accurate infection level impossible.

So, we consider the following modification of the model of section 3.4:

$$p_x^f = f p_x \tag{3.43}$$

where p_x^f is the available fake information of the agents and f is a coefficient indicating its deviation from the actual information p_x. So, we get the following simulations (Figure:3.14) indicating the effects of an overestimation of the infection level ($f = 2$) and an underestimation of the infection level ($f = 0.5$), in comparison with the game with actual information. We observe that in the case of an overestimation of the infection level the agents care more to

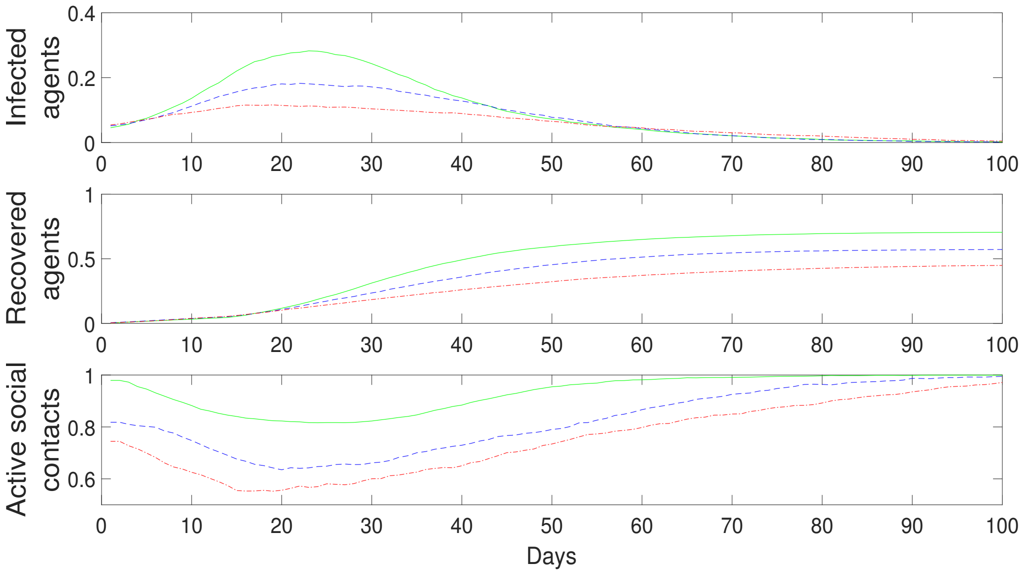

Figure 3.14 Infection, recovery and sociability curves for games with fake statistical information. The fake information coefficient is $f = 0.5$ (solid line), $f = 1$ (dashed line) and $f = 2$ (dashed-dotted line).

follow social distancing and the disease prevalence is kept at low levels, while in the case of underestimation of the infection the agents do not care so much and the disease prevalence is higher. In Figure 3.15 we point out the negative correlation of the infection outspread with the fake information coefficient (f).

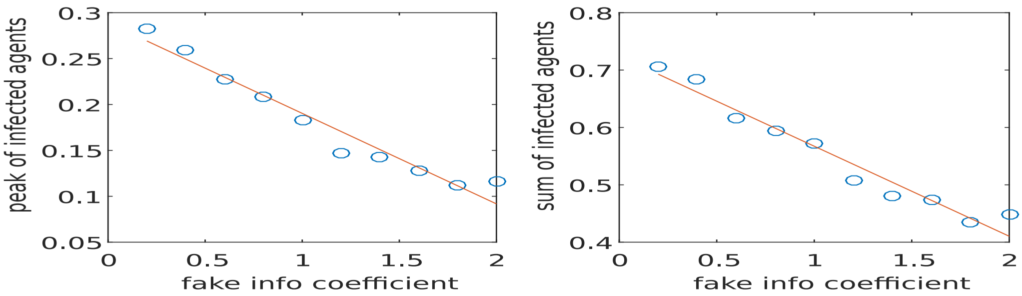

Figure 3.15 Correlation of the coefficient of fake information (f) with the infection outspread

3.7 Conclusion

A game-theoretic approach of social distancing has been considered. For the game model introduced the Nash equilibria are computable and we propose algorithms to find them. So, when the agents follow the Nash equilibrium strategies, we investigate the effects of spontaneous social distancing on the prevalence of the epidemic, both analytically and numerically through simulations on artificial networks. We study also numerically the role of the networked structure of human interconnections and of the available information on the agents behaviour and on the epidemic's outspread.

Future work in this direction may include the following. At first, in the game analysed in Section 3.4 may arise more equilibria, so it is interesting, if possible, to characterize all the equilibria. Secondly, the study of the case of a social planner making infrastructure modifications, e.g. cancelling flights, that affect the topology of the graph and thus the action space and the behavior of the agents.

Chapter 4

Almost-Sure Finite-Time Stochastic Min-Max Consensus

4.1 Introduction

Consensus protocols have gained significant attention in the last decades [94], as they have become an integral part of many decentralised systems' tasks. Their applications vary from information fusion, averaging, coordination and formation control to decentralised optimisation and decision making. For all the aforementioned heterogeneous applications, several variations of consensus protocols have been developed, such as weighted average consensus and minimum-maximum consensus [106, 92, 89].

Due to recent advances in communication technologies and embedded systems, the application of distributed control systems vary from computer networks, smart grids and sensor networks to networked cyber-physical systems. In parallel to this expansion, new challenges have arisen for consensus protocols concerning security and privacy issues, since cyber-physical systems are prone to attacks from malicious agents [88]. To deal with these issues, some recent approaches focus on developing privacy-preserving consensus protocols [102, 82], using methods from cryptography and attack detection.

While being well-established, these methods have running-time, bandwidth and energy consumption requirements that may deter from their implementation on autonomous agents with limited resources. Furthermore, by definition, average consensus protocols have asymptotic, and therefore infinite-time, convergence, which restricts their applicability on tasks demanding both high-precision and fast convergence. The above discussion shows that the implementation of a finite-time, privacy preserving consensus protocol is a challenging problem.

To the best of our knowledge, in the existing literature on finite-time consensus and control of dynamic agents, e.g. [80, 79, 120, 127, 56], the agents considered are of first or second-order dynamics and use continuous measurements of the relative differences between the states of their neighbors. Sampled neighbor data have been used in [81] for simple-integrator agents, where the authors consider a finite-time, event-triggered, deterministic control strategy, that assumes fixed topology having a spanning tree.

Current approaches on stochastic protocols, e.g. [113, 118, 109, 63], focus on modelling and handling the stochastic disturbances of the communication procedure and/or the stochastic switches in the communication topology [119, 61]. Moreover, the authors of [126] consider consensus under stochastic sampling. Recently, an interesting idea has been introduced in [69], where the agents stochastically choose the time instants to exchange information in order to counter jamming attacks.

Motivated by these approaches, on the first part of our work, we propose a finite-time, min-max consensus algorithm with stochastic mixing. We prove that the protocol achieves finite-time consensus to a random value, which ensures that the consensus value is impossible to be intercepted by a curious (malicious) attacker, who eavesdrops the network during the transient state of the protocol. Additionally, it does not need extra computational resources for encryption or attack detection. On the second part of our work, we propose a distributed control law for continuous-time high-order agents, based on the stochastic mixing protocol of the first part. We introduce new continuous auxiliary variables, that utilize the stochastic protocol, and employ only samples of the neighbors' outputs, with an arbitrarily large sampling period. The variables reformulate the finite-time consensus problem to the finite-time regulation problem for the new variables, which is then solved by a classical finite-time control algorithm [14]. Our methodology overcomes the limitations of fast sampling [64], that may occur by an event-triggering mechanism, works on high-order agent dynamics with switching, not necessarily connected topologies, and its stochastic nature makes it suitable for applications related to security.

The rest of the paper is organized as follows. In Section 4.2, we include some preliminaries from graph theory and the min-max consensus protocol. In Section 4.3, we introduce the new protocol and we prove that it converges almost-surely in finite-time. In Section 4.4, using the protocol and introducing suitable auxiliary variables, we design a new finite-time consensus control law for a swarm of autonomous agents in integrator chain form. Finally, in Section 4.5, we present simulations that demonstrate the validity of the preceding analysis.

4.2 Preliminaries

4.2.1 Graph Theory

A directed graph $G = (V, E)$ consists of a finite set of nodes V and a set of arcs $E = \{e_{ij} = (i, j)\} \subseteq V \times V$. We denote $N_i = \{j, e_{ji} \in E\}$ the neighborhood of node i and $\bar{N}_i = N_i \cup \{i\}$. A successive sequence of nodes and pairwise distinct arcs is a path. A node j is said to be reachable from i if there exists a path from i to j. A digraph G is called strongly connected if any node i is reachable from any other node j. If $S \subset V$, the cut induced by S is the set of arcs from S to $V \setminus S$, i.e. the set of arcs leaving S. The capacity $c(S)$ of a cut is the number of its arcs. We denote the maximum capacity over all the possible cuts of a digraph by $c_{\max}(G)$. The union of two digraphs with the same node set $G_1 = (V, E_1)$, $G_2 = (V, E_2)$ is defined as $G_1 \cup G_2 = (V, E_1 \cup E_2)$. A sequence of graphs $\{G_k\}_k$ is called Uniformly Jointly Strongly Connected (UJSC) [106], if there exists some integer $B \geq 1$ such that the unions of digraphs $U_l = \bigcup_{k=l}^{l+B-1} G_k$ is strongly connected $\forall l \in \mathbb{N}^* := \mathbb{N} \setminus \{0\}$.

4.2.2 Max (Min) Consensus Protocol

We consider a network of agents represented by the nodes $V = \{1, \ldots, N\}$. The outputs are assumed to be sampled with sampling period T_s. Each agent i has a scalar output $y_i[k] = y_i(kT_s)$ at each time step k. At each time instant the interactions among the agents are modeled by a digraph $G_k = (V, E_k)$.

Assumption 1. *We assume that the sequence of graphs $\{G_k\}_k$ is UJSC, for some integer B.*

Definition 1. *The agents achieve finite-time consensus for initial condition $y[k_0] = y^0 \in \mathbb{R}^N$ if there exist a $y^*(y^0) \in \mathbb{R}$ and an integer $T(y^0) > k_0$ such that $y_i[k] = y^*$, for all $i = 1, \ldots, N$, for all $k \geq T$. Global finite-time consensus is achieved if finite-time consensus is achieved for all $y^0 \in \mathbb{R}$ and $k_0 \in \mathbb{N}$.*

We recall, at first, the following Max consensus protocol from [106], [92]:

$$y_i[k+1] = \max\{y_j[k], \quad j \in \bar{N}_i[k]\}. \tag{4.1}$$

Proposition 1. *If Assumption 1 holds and $y_i[k]$ evolves according to* (4.1)*, then the states of all the agents converge to a consensus value $y^* = y_0^M$, which is the maximum among $y_i[0]$, in at most $\Delta = B(N-1)$ steps.*

Proof: An agent with maximum state will retain its state due to (1). We consider the set of nodes $V_M[k]$ of all agents having the maximum state $y^M[k] = \max\{y_i[k], i \in V\} =$

$\max\{y_i[0], i \in V\} = y^M[0]$. As a result, the index set V_M is increasing. Since Assumption 1 holds, for every $n = 1, \dots, N-1$ there exists at least one agent j and a time $k_j \in \{(n-1)B + 1, \dots, nB\}$ such that $V_M[k_j] \cap N_j[k_j] \neq \emptyset$, since the capacity of the minimum cut is greater than or equal to one due to strong connectivity, and thus $V_M[k_j] \cup \{j\} \subseteq V_M[k_j+1]$. Hence, $V_M[B(N-1)] = V$, which concludes the proof.

Remark 1. *The same analysis holds for the Min consensus protocol* $y_i[k+1] = min\{y_j[k], \quad j \in \bar{N}_i[k]\}$*, which converges to* $y^* = min\{y_i[0], i \in V\}$ *in at most* $\Delta = B(N-1)$ *steps.*

4.3 Stochastic Min-Max Consensus Protocol

We propose the following stochastic min-max consensus protocol:

$$\begin{aligned} y_i[k+1] =& \lambda_i[k] \min\{y_j[k], \quad j \in \bar{N}_i[k]\} \\ &+ (1-\lambda_i[k]) \max\{y_j[k], \quad j \in \bar{N}_i[k]\}. \end{aligned} \tag{4.2}$$

The weights $\lambda_i[k] \in [0,1]$ are given by:

$$\lambda_i[k] = \max\{0, \min\{1, w_i[k]\}\} \tag{4.3}$$

where for each time step k, $\forall i = 1, \dots, N$, $w_i[k]$ are independent identically distributed (i.i.d.) random variables, following a distribution F_k^w on a set $A_k \subseteq \mathbb{R}$ such that $(A_k, \mathcal{B}(A_k), F_k^w)$ is a probability space, where $\mathcal{B}(A_k)$ is the Borel σ-algebra on A_k; $A_k = A_k^l \cup [0,1] \cup A_k^r$ with $A_k^l \subseteq (-\infty, 0)$, $A_k^r \subseteq (1, +\infty)$ and $P_{F_k^w}(A_k^l \cup A_k^r) > 0$.

Remark 2. *Due to the randomness in the selection of the weights* λ_i*, this protocol guarantees consensus to some random point within* $[\min_{i \in V} y_i[0], \max_{i \in V} y_i[0]]$. *This property enhances security, in the sense that an eavesdrop attacker cannot intercept the final consensus value, even if he has information about the state of the system and the algorithm at some time.*

4.3.1 Convergence of the stochastic protocol (4.2) in finite-time

We observe that if $w_i[k] \leq 0$ then $y_i[k+1] = \max\{y_j[k], \quad j \in \bar{N}_i[k]\}$, else if $w_i[k] \geq 1$ then $y_i[k+1] = \min\{y_j[k], \quad j \in \bar{N}_i[k]\}$. Let us now define:

$$p[k] := P(w_i[k] \leq 0) = P_{F_k^w}(A_k^l) \tag{4.4}$$

$$p'[k] := P(w_i[k] \geq 1) = P_{F_k^w}(A_k^r) \tag{4.5}$$

and denote $p_1 = p[1]$.

Assumption 2. *We assume that at least one of the sequences $(p[k])_k$, $(p'[k])_k$ is non-decreasing.*

Example 1. *If the distribution F_k is the uniform distribution on $[-\delta, 1+\delta]$ for every k then $p[k] = p'[k] = \frac{\delta}{1+2\delta}$.*

Lemma 1. *If Assumptions 1 and 2 hold, then the stochastic protocol* (4.2) *achieves consensus in a time window of Δ consecutive steps $k = l, ..., l+\Delta$, to the maximum (or minimum) value of the agents states at time $k = l$, with probability:*

$$p_c[l] \geq \min_{(k_1,...,k_N)\in K} \prod_{j=1}^{N-1} (p[l+k_j])^{\min\{i,C_M\}(k_{j+1}-k_j)+1} \tag{4.6}$$

where $K = \{(k_1, ..., k_N) : k_j \in \mathbb{N}, k_1 = 0 \leq \cdots \leq k_j \leq k_{j+1} \leq \cdots \leq k_N = \Delta\}$ and $C_M = \max_k\{c_{\max}(G_k)\}$ for all $k \in \mathbb{N}$.

Proof: Without loss of generality, from Assumption 2, we consider the case that $p[k]$ is non-decreasing. Since Assumption 1 holds, we use a similar argument with that of the proof of Proposition 1 on a union of Δ graphs $U_l := \bigcup_{k=l}^{l+\Delta} G_k$, where $\Delta = B(N-1)$. Specifically, we consider the worst case scenario that only a single node i_1 has the maximum value $y_{i_1}[l]$ at the time instant $k = l$ and a spanning tree of maximal depth Δ from this node containing all the other nodes $i_1, ..., i_j, ..., i_N$. We then consider the sets $V_M[k_j] = \{i_1, ..., i_j\}$, where k_j is the minimum time step that i_j is reachable from i_1 on the graph $U_{l,k_j} := \bigcup_{k=l}^{l+k_j} G_k$ and $k_1 = 0$. Assume that the agents in $V_M[k_j]$ have the maximum value, $y_m[l+k_j] = y_{i_1}[l]$, $m = 1, ..., i_j$.

If for all times $k = l+k_j+1, ..., l+k_{j+1}$, for all $j = 1, ..., N$, all the agents in $V_M[k_j]$ hold their values and at $k = l+k_{j+1}$ the agent i_{j+1} chooses the maximum value of its neighbors, then $V_M[k_1] \subseteq V_M[k_2] \subseteq ... \subseteq V_M[\Delta]$ and thus the stochastic protocol converges.

For every time window $k = l+k_j+1, ..., l+k_{j+1}$, we consider the event E_j^1: every agent that communicates with agents in the set $V \setminus V_M[k_j]$ to follow the max protocol in this time window and thus holds its maximum value. Defining the sets $V_M^{out}[k] = \{m : m \in V_M[k_j], N_m[k] \setminus V_M[k_j] \neq \emptyset\}$, the probability of the event E_j^1 can be expressed as $P(E_j^1) = \prod_{k=l+k_j+1}^{l+k_{j+1}} \prod_{m\in V_M^{out}[k]} (p[k])^{|N_m[k]\setminus V_M[k_j]|}$. Since $|N_m[k] \setminus V_M[k_j]| \leq \min\{j, c_{\max}(G_k)\}$ for all $k = l+k_j+1, ...l+k_{j+1}$, $m = i_1, ..., i_j$ and $(p[k])_k$ is non-decreasing, $P(E_j^1) \geq p[l+k_j])^{\min\{j, C_M^{l+k_j+1, l+k_{j+1}}\}(k_{j+1}-k_j)}$, where $C_M^{l+k_j+1, l+k_{j+1}}$ is the maximum capacity of G_k for $k = l+k_j+1, ..., l+k_{j+1}$. We consider also the event E_j^2: the agent i_{j+1} to follow the max protocol at $l+k_{j+1}$, with probability $P(E_j^2) = p[l+k_{j+1}]$. The events E_j^1 and E_j^2 are independent, hence the probability of the event $E_j = E_j^1 \cap E_j^2$ has the following bound:

$$P(E_j) \geq (p[l+k_j])^{\min\{j, C_M^{l+k_j+1, l+k_{j+1}}\}(k_{j+1}-k_j)+1}. \tag{4.7}$$

For the probability p_{U_l} for the algorithm to converge over a topology $U_l = \bigcup_{k=l}^{l+\Delta} G_k$ which induces the times k_j, it holds that $p_{U_l} \geq \prod_{j=1}^{N-1} P(E_j)$. From (4.7) we get:

$$p_{U_l} \geq \prod_{j=1}^{N-1} (p[l+k_j])^{\min\{j, C_M^{l,l+\Delta}\}(k_{j+1}-k_j)+1}$$

where $C_M^{l,l+\Delta}$ is the maximum capacity of G_k for $k = l, ..., l+\Delta$. Finally, the probability that the algorithm converges on any $U_l = \bigcup_{k=l}^{l+\Delta} G_k$ is

$$p_c[l] \geq \min_{(k_1,...,k_N) \in K} \prod_{j=1}^{N-1} (p[l+k_j])^{\min\{i, C_M\}(k_{j+1}-k_j)+1}$$

where $K = \{(k_1, ..., k_N) : k_1 = 0 \leq \cdots \leq k_j \leq k_{j+1} \leq \cdots \leq k_N = \Delta\}$ and $C_M = \max_{k \in \mathbb{N}}\{c_{\max}(G_k)\}$.

Using Lemma 1 the following proposition can be proved.

Proposition 2. *If Assumptions 1 and 2 hold, then the probability $p_c[l]$ for the stochastic protocol* (4.2) *to achieve consensus in a time window of Δ consecutive steps $k = l, \ldots, l+\Delta$, to the maximum (or minimum) value of the agents states, is bounded by:*

$$p_c[l] \geq (p[l])^{1+C_M(\Delta-1)} \tag{4.8}$$

Proof: From Assumption 2 and without loss of generality, we consider again the case that $p[k]$ is non-decreasing, i.e. $p[l+k_j] \geq p[l]$ for all k_j. Therefore, we obtain the following bounds for $p_c[l]$, defined in Lemma 1:

$$\begin{aligned} p_c[l] &\geq \min_{(k_1,...,k_N) \in K} \prod_{j=1}^{N-1} (p[l+k_j])^{\min\{j, C_M\}(k_{j+1}-k_j)+1} \\ &\geq \min_{(k_1,...,k_N) \in K} \prod_{j=1}^{N-1} (p[l])^{\min\{j, C_M\}(k_{j+1}-k_j)+1} \geq (p[l])^{M(K)} \end{aligned} \tag{4.9}$$

where $M(K) = 1 + \max_{(k_1,...,k_N) \in K}(\sum_{j=1}^{C_M-1} i(k_j - k_{j-1}) + C_M \sum_{j=C_M}^{N-1}(k_{j+1} - k_j))$. Defining $x_1 = k_1$ and $x_j = k_{j+1} - k_j$, we have that $x_j \geq 0$ for all $j = 1, \ldots, N-1$ and $\sum_{j=1}^{N-1} x_j = k_{N-1} \leq \Delta - 1$ from the definition of k_j's. The maximum exponent in (4.9) can be found by solving the following integer optimization problem.

$$M(x) = 1 + \max_x \Big(\sum_{j=1}^{C_M - 1} j x_j + C_M \sum_{j=C_M}^{N-1} x_j \Big) \tag{4.10}$$

$$\sum_{j=1}^{N-1} x_j \leq \Delta - 1 \tag{4.11}$$

Since the coefficients of the linear objective function (4.10) are positive and the maximum coefficient is C_M, the solution of the integer optimization problem (4.10)-(4.11) can be computed by choosing $x_j = 0$ for $j = 1, \ldots, C_M - 1$ and concentrating all the mass of x_j's, derived from (4.11), on the second term of the objective function: $M^* := M(x^*) = 1 + C_M(\Delta - 1)$. Hence $p_c[l] \geq (p[l])^{M^*}$ which concludes the proof.

Consider $T > \Delta$ iterations of the protocol (4.2) and let us define the sequences of length Δ of random vectors:

$$S_l = (w[l], \ldots, w[l+\Delta]), \quad l = 1, \ldots, T - \Delta + 1 \tag{4.12}$$

where $w[k] = (w_1[k], ..., w_N[k])$ are independent random vectors. Let $\mathcal{L}(T)$ be the set of all these sequences and $L(T) = T - \Delta + 1$ be its size. These sequences may be dependent in pairs, for example the pairs (S_m, S_n) with $m < n < m + \Delta$ that share a common part $(w[n], ..., w[m+\Delta])$. Due to this fact, in order to proceed with our analysis, we consider the set $\mathcal{L}_I(T) := \{S_{l_i} : l_i = i\Delta + 1, i = 0, \ldots, L_I(T)\}$, where the size of this set $L_I(T) = \lfloor T/\Delta \rfloor$ is the largest integer smaller than or equal to T/Δ. The sequences $S_l \in \mathcal{L}_I(T)$ are independent since they do not overlap by definition and the random vectors $w[k]$ are independent.

Theorem 1. *If Assumptions 1 and 2 hold, then the stochastic protocol* (4.2) *achieves global finite-time consensus with probability 1 (almost surely).*

Proof: Taking into account Assumption 2 and without loss of generality, we consider the case that $p[k]$ is non-decreasing. From Lemma 1 we have that the probability for the stochastic protocol (4.2) to converge in a time window $(l, \ldots, l + \Delta - 1)$ is $p_c[l]$, as defined in (4.6). Moreover, since $\mathcal{L}_I(T) \subset \mathcal{L}(T)$, the probability that such a sequence does not exist in $\mathcal{L}(T)$ is smaller than the probability of non existence in $\mathcal{L}_I(T)$. Considering the event $E^c_{\mathcal{L}_I} = \{$no max consensus in $S_{l_i}, \forall S_{l_i} \in \mathcal{L}_I(T)\}$, its probability is

$$P(E^c_{\mathcal{L}_I}) = \prod_{l_i=1}^{L_I(T)} (1 - p_c[l_i]) \leq (1 - p_1^{M^*})^{L_I(T)} \tag{4.13}$$

since the sequences in $\mathcal{L}_I(T)$ are independent and $p_c[l_i] \geq (p[l_i])^{M^*} \geq p_1^{M^*}$, $\forall l_i$, from Proposition 2. Thus, the probability of a sequence $S \in \mathcal{L}(T)$ where the protocol (4.2) converges is greater than $1-(1-p_1^{M^*})^{L_I(T)}$. Let us now define the random variables

$$X_T = \begin{cases} 1 & \text{,if (4.2) converges in } T \text{ steps} \\ 0 & \text{,else} \end{cases} \tag{4.14}$$

and

$$X_T' = \begin{cases} 1 & \text{,if } \exists\, S_l \in \mathcal{L}(T) \text{ s.t. max consensus} \\ 0 & \text{,else} \end{cases} \tag{4.15}$$

For every realization ω of the random variables $w_i[k]$, $i = 1,...,N$, $k = 1,...,T$, it holds that $X_T(\omega) \geq X_T'(\omega)$, since the stochastic protocol (4.2) could also converge if there exists a sequence which results in minimum consensus. Hence,

$$P(X_T = 1) \geq P(X_T' = 1) \geq 1-(1-p_1^{M^*})^{L_I(T)}. \tag{4.16}$$

We define now the random variable:

$$X_\infty = \begin{cases} 1 & \text{,if (4.2) converges in finite steps} \\ 0 & \text{,else} \end{cases} \tag{4.17}$$

For every realization ω of the random variables $w_i[k]$, $i = 1,...,N$, $k = 1,...,T$, it holds that $X_T(\omega) \leq X_{T+1}(\omega)$, $X_T(\omega) \leq X_\infty(\omega)$ for all T and $X_T(\omega) \uparrow X_\infty(\omega)$ as $T \to +\infty$. Invoking the Monotone Convergence Theorem and (4.16) we get that

$$\begin{aligned} P(X_\infty = 1) &= \lim_{T\to+\infty} P(X_T = 1) \\ &\geq \lim_{T\to+\infty} \left(1-(1-p_1^{M^*})^{L_I(T)}\right) = 1 \end{aligned} \tag{4.18}$$

since $p_1 \in (0,1]$ for all T, which concludes the proof.

4.4 Finite time consensus of integrator chains

We consider N agents in integrator chain form:

$$\begin{aligned} \dot{x}_{i,j} &= x_{i,j+1},\ (j = 1,\dots,m-1) \\ \dot{x}_{i,m} &= u_i,\ y_i = x_{i,1}. \end{aligned} \tag{4.19}$$

Our objective is to design a distributed control law for each agent i, using only samples of the neighbors' outputs, based on the protocol (4.2), in order to achieve output consensus in finite-time. The outputs of each neighbor of agent i are sampled every T time instants. We define:

$$\begin{aligned} s_i[k] =& \lambda_i \min\{y_j(kT), j \in \bar{N}_i(kT)\} \\ &+ (1-\lambda_i)\max\{y_j(kT), j \in \bar{N}_i(kT)\} \end{aligned} \tag{4.20}$$

These variables are discontinuous, since they are updated every sampling period. Therefore, one cannot directly use them as references for the virtual control laws for the system of agents (4.19). To ensure that the resulting virtual control laws are continuously differentiable whenever a new sample is added, we employ the following m-times continuously differentiable function $q : [0,\infty) \to [0,1]$:

$$q(t) = \begin{cases} 1 & \text{, if } t > T \\ q_m \int_0^t \left(\sin\left(\frac{\pi\sigma}{T}\right)\right)^{2m} d\sigma & \text{, if } t \in [0,T) \end{cases} \tag{4.21}$$

with $q_m := (2^m m!)^2/[(2m!)T]$. For our distributed control design we define the variables

$$\begin{aligned} \eta_i(t) :=& y_i(t) - q(t-kT)s_i[k] \\ &- [1-q(t-kT)]s_i[k-1] \end{aligned} \tag{4.22}$$

where the function $q(\cdot)$ is given by (4.21) and $k = \left\lfloor \frac{t}{T} \right\rfloor$. These new variables are m-times continuous differentiable for continuous inputs u_i, since $\lim_{t\to kT^-} \eta_i(t) = y_i(kT) - s_i[k-1] = \lim_{t\to kT^+} \eta_i(t)$ and $\lim_{t\to kT^-} \dot{\eta}_i^{(j)}(t) = \lim_{t\to kT^+} \dot{\eta}_i^{(j)}(t) = y_i^{(j)}(kT)$, due to the fact that $q^{(j)}(0) = q^{(j)}(T) = 0$. Then, we consider the error variables $z_{i,1} = \eta_i$, $z_{i,2} = \dot{\eta}_i$,..., $z_{i,m} = \eta_i^{(m-1)}$ and their dynamics:

$$\begin{aligned} \dot{z}_{i,j} &= z_{i,j+1}, \ (j = 1,\ldots,m) \\ \dot{z}_{i,m} &= u_i - q^{(m)}(t-kT)(s_i[k] - s_i[k-1]). \end{aligned} \tag{4.23}$$

We design the input for system (4.23) as follows:

$$u_i = q^{(m)}(t-kT)(s_i[k] - s_i[k-1]) + \bar{u}_i \tag{4.24}$$

where

$$\bar{u}_i = -k_1 \mathrm{sign}(z_{i,1})|z_{i,1}|^{\alpha_1} - \cdots - k_m \mathrm{sign}(z_{i,m})|z_{i,m}|^{\alpha_m}. \tag{4.25}$$

Coefficients $k_1, \dots, k_m > 0$ are selected such that the polynomial $s^m + k_m s^{m-1} + \cdots + k_2 s + k_1$ is Hurwitz and $\alpha_1, \dots, \alpha_m$ satisfy $\alpha_{i-1} = \frac{\alpha_i \alpha_{i+1}}{2\alpha_{i+1} - \alpha_i}$, with $\alpha_{m+1} = 1$, $\alpha_m = \alpha$ and $\alpha \in (1-\varepsilon, 1)$, for some ε. In [14] the authors show that there exists some ε such that $\bar{u}_i$ stabilizes a chain of integrators to the origin in finite-time. Based on this result, we prove the following theorem.

Theorem 2. *If Assumptions 1 & 2 hold, the multi-agent system with agents dynamics defined by* (4.19) *achieves finite-time output consensus under the decentralized control protocol* (4.24), (4.25). *Moreover, all the signals of the closed loop system remain uniformly bounded.*

Proof: From proposition 8.1 of [14], we deduce that the states of the system (4.23) converge to the origin in finite-time. Therefore, the state $z_{i,1} = \eta_i$ is uniformly bounded by a constant H_i for each agent i. Let $H = \max\{H_i\}$ be the maximum among these bounds. We denote T_i^c the finite-time of convergence for the dynamics (4.23) of agent i and $T_M^c = \max\{T_i^c\}$ the maximum convergence time. Moreover, from (4.20), we observe that $\max\{|s_i(kT)|\} \leq \max\{|y_i(kT)|\}$ and we denote by $Y_k = \max_i\{|y_i(kT)|\}$ the maximum absolute value of the outputs at time kT. We rewrite (4.22):

$$\begin{aligned} y_i(t) =& \eta_i(t) + q(t-kT)\, s_i(kT) \\ &+ [1 - q(t-kT)]\, s_i((k-1)T) \end{aligned} \tag{4.26}$$

and we observe that for $t = T$: $Y_1 = \max_i\{|y_i(T)|\} \leq H + Y_0$, where $Y_0 = \max_i\{|y_i(0)|\}$ is the maximum among the initial conditions. Recursively, from (4.26), we obtain that:$Y_k = \max\{|y_i(kT)|\} \leq kH + Y_0$ and for $K_1 = \lceil \frac{T_M^c}{T} \rceil$, which is the time that all η_i have converged to zero, $Y_{K_1} \leq K_1 H + Y_0$. After that time, the outputs of the agents $y_i(kT)$ follow the stochastic protocol (4.2) with zero error $\eta_i(t)$. Thus, from Theorem 1, there exists almost surely a finite-time step K_2 such that $y_i(kT)$ reach consensus for all $k > K_2$. From (4.20), $s_i(kT) = y_i(K_2 T), \forall k > K_2$ and therefore from (4.26) $y_i(t) = y_i(K_2 T), \forall t > (K_2+1)T$.

4.5 Simulations

We consider a network of five agents with a switching ring topology, as depicted in Fig.4.1. The agents' dynamics are considered to be double integrators. We apply the control protocol (4.24) to each agent. The samples $s_i[k]$ follow the stochastic consensus protocol (4.20), with $P(w_i \leq 0) = P(w_i \geq 1) = 0.3$ and λ_i is uniformly distributed in the interval $(0,1)$ with $P(w_i \in (0,1)) = 0.4$, for all time steps k. In Fig.4.2, a simulation of the agents outputs and their control inputs is presented. Fig.4.3 shows a histogram of the points of convergence, which confirms the stochastic nature of the protocol and a histogram of the convergence times, which highlights the fast convergence of the protocol.

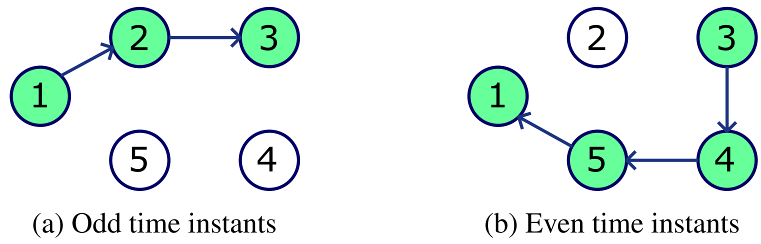

Figure 4.1 Graph topology

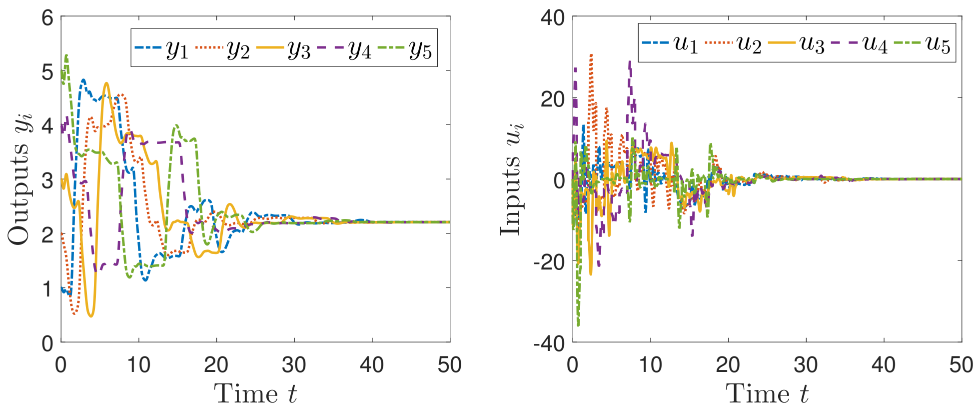

Figure 4.2 Outputs y_i and inputs u_i

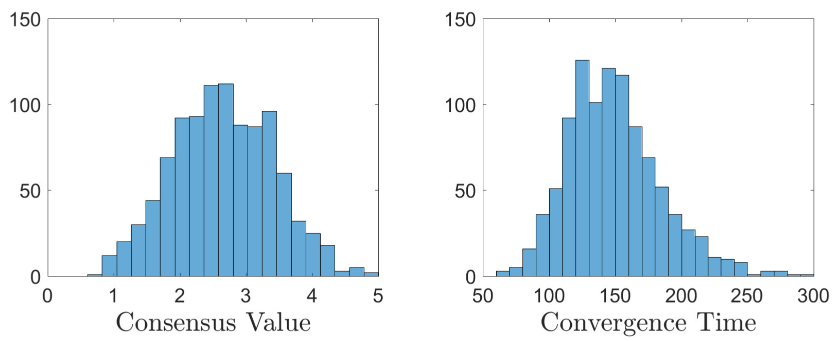

Figure 4.3 Histogram of consensus points and convergence times

4.6 Conclusion

A novel stochastic min-max consensus protocol and a control law are introduced, which ensure finite-time consensus of multi-agent systems with high-order dynamics. This protocol, requires minimal information exchange among the agents (only samples of the outputs) and is applicable in networks with switching topology.

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Curriculum Vitae

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Education

2016-2022	**PhD Candidate at School of Electrical & Computer Engineering, NTUA** Subject: "Game Theory and Dynamic Mechanisms on Graphs" Supervisor: Prof. George P. PAPAVASSILOPOULOS
2011-2016	**School of Electrical & Computer Engineering** Grade: 9.34/10

Work experience

1/2/2021 - 30/6/2022	**Project Manager at Glafcos Marine Ltd** Head of the R&D department, managing 3 Horizon2020 research projects, 2 EPAnEK-ESPA 2014-2020 research projects and several in-house projects in robotics and marine applications.
1/9/2022 - Now	**Researcher at SmartRue, ICCS** Managing SYNERGIES, a Horizon Europe research project and working as a researcher.

Projects

1. ROBINS: ROBotics technology for INspection of Ships, 1 January 2018 - 30 June 2021, https://www.robins-project.eu/

2. BugWright2: Autonomous Robotic Inspection and Maintenance on Ship Hulls, 1 January 2020 - 31 March 2024, https://www.bugwright2.eu/

3. GATERS: Design, Manufacture and Installation of a retrofit Gate Rudder System (GRS), 1 February 2021 - 31 January 2024, https://www.gatersproject.com/

4. SOUP: Soilless Culture Upgrade, 1 June 2018 - 31 May 2022, https://soup-project.gr/

5. Wave2energy: Energy from the Waves, Innovative Technology for Direct and Indirect Utilization, 1 June 2020 - 31 May 2023

6. SYNERGIES: Shaping consumer-inclusive data pathwaYs towards the eNERGy transItion, through a reference Energy data Space implementation, 1 September 2022 - 28 February 2026, https://synergies-project.eu/

Publications List

Journal Papers

1. A.-R. Lagos, G. P. Papavassilopoulos "Network Topology Design to Influence the Effects of Manipulative Behaviors in a Social Choice Procedure" Journal of the Franklin Institute 359 (7), 3046-3070

2. A.-R. Lagos, I. Kordonis, G.P. Papavassilopoulos, "Games of social distancing during an epidemic: Local vs statistical information", Computer Methods and Programs in Biomedicine Update, Vol. 2, 2022, 100068, ISSN 2666-9900.
3. Athanasios-Rafail Lagos, Harris E. Psillakis, Athanasios K. Gkesoulis “Almost-Sure Finite-Time Stochastic Min-Max Consensus” (Under revision in IEEE Transactions on Circuits and Systems)
4. I. Kordonis, A.-R. Lagos, G. P. Papavassilopoulos “Dynamic Games of Social Distancing during an Epidemic: Analysis of Asymmetric Solutions” Dynamic Games and Applications (2022) 12:214–236
5. Kordonis, Ioannis, Athanasios-Rafail Lagos, and George P. Papavassilopoulos. "Nash social distancing games with equity constraints: How inequality aversion affects the spread of epidemics." Applied Mathematics and Computation 434 (2022): 127453.
6. A. K. Gkesoulis, H. E. Psillakis and A. -R. Lagos, "Optimal Consensus via OCPI Regulation for Unknown Pure-Feedback Agents With Disturbances and State Delays," in IEEE Transactions on Automatic Control, vol. 67, no. 8, pp. 4338-4345, Aug. 2022, doi: 10.1109/TAC.2022.3179218.
7. Haris E. Psillakis, Athanasios K. Gkesoulis, Athanasios-Rafail Lagos “Adaptive NN Resilient Consensus for High-Order Continuous Nonlinear Agents Using Delayed Neighbor Samples” (Under preparation)
8. Psillakis, H.E. and Lagos, A-R “Unifying adaptive control with the nonlinear PI methodology: Designs for unknown strict-feedback nonlinear systems with non-smooth actuator nonlinearities”, International Journal of Adaptive Control and Signal Processing. Article DOI: 10.1002/acs.2846
9. Spiros Argyros, Alexandros Georgiou, Athanasios-Rafail Lagos and Pavlos Motakis “Joint spreading models and uniform approximation of bounded operators”, Studia Mathematica, December 2017

Conference Papers

1. A-R. Lagos, G. P. Papavassilopoulos, “Self-Consistent and Manipulative Behaviour in Social Choice as a Repeated Nash Game on a Graph”, 42nd Annual Meeting of the AMASES, Naples, Italy, 13-15 September 2018.

2. Ioannis Kordonis, Athanasios-Rafail Lagos, George P. Papavassilopoulos, "Game-Theoretic Modeling of Social Distancing: Analysis of Case Studies", Workshop on Dynamic Games and Applications, Paris, October 27-28, 2022.

Teaching Activities

- 1 semester teaching assistance in the graduate course “Stochastic Optimization” in Electrical and Computer Engineering Department of NTUA.
- 4 years occupation in private teaching of undergraduate courses (mainly math courses).
- 4 years occupation in private teaching in secondary education.
- 2 years occupation in a social institute for reinforcement teaching.

Fellowships/Prizes

- 2016, fellowship of ELKE (special foundation for research and novelty) for the dissertation of the PhD.
- 2011, fellowship of IKY (national institute of fellowships) for becoming accepted in the third position in the school of Electrical and Computer Engineering.
- 2011, fellowship of Eurobank for graduating first among the graduating class from high school (Valedictorian award).

Languages spoken

- Greek (Native)
- English (Proficiency certification from Michigan University - C2 European level)
- French (Delf - B2 European level)